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This study investigated the mechanism of stress and deformation in particulate materials under repeated application of moving surface loads. It consisted of two complimentary phases: the theoretical development of a model to characterize the propagation of stress and deformation in the medium and the experimental investigation of its adequacy.

The research project built on the earlier work of Bourdeau, who brought together the probabilistic concept of stress transmission in granular bodies (Harr) and the analogous stochastically based deformation mechanism proposed by Litwiniszyn, to investigate the response of particulate material to vertically applied slowly moving surface loadings. The perceived advantage of this methodology is that the response of granular materials may be characterized by a very few parameters, whereas the older stress-dilatancy and yield theories abound in extra parameters.

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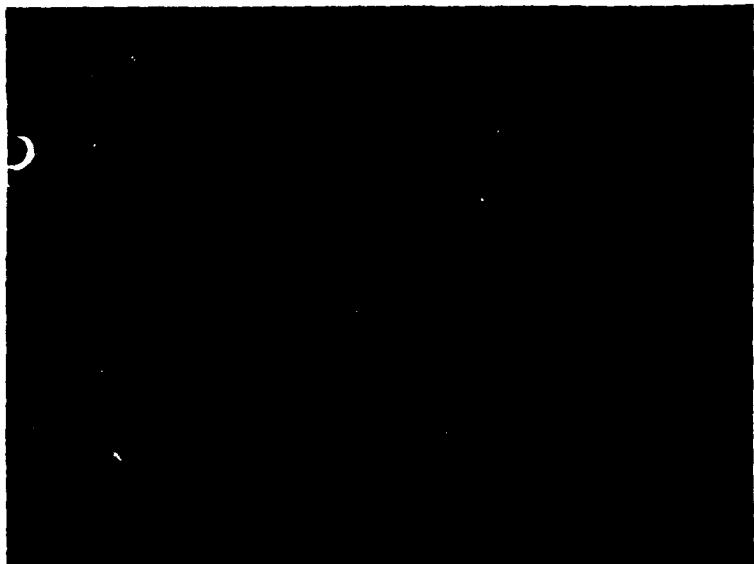
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The testing phase was undertaken at the soil mechanics laboratory of the Swiss Federal Institute of Technology (EPFL) at Ecublens, Switzerland. A fine dry silt soil was tested under both repeated static loading and quasi-mobile loading. An array of lead shot was placed periodically throughout the soil body, and records were made of the deformation of the soil under the surface applied loading through exposure of the soil model to X-rays. Load was applied to the surface of the soil through a rigid rough plate. In the static series of loads, the plate was repeatedly applied to the same point on the soil surface, whereas in the moving load series, the load plate was relocated contiguous to its prior position at each application. In this way, for the latter, a "quasi-moving" load mechanism was modeled.

The conclusion was drawn that the response of dry granular material to vertically applied moving loads is one-dimensional, and the magnitude of deformation may be estimated from an equivalent static loading. There is evidence that Harr's coefficient of lateral stress ( $v$ ) is not constant throughout the material under load but varies significantly with depth. The pattern of the variation of  $v$  with depth under the load is consistent with Harr's modification of Schmertmann's method of settlement analysis and lends credence to it.

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FINAL REPORT

Mechanics of Soils as Particulate  
Media Under Moving Loads

Contract (F49620-87-K-0006)

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## ABSTRACT

The objective of this research project was to build on the earlier work of BOURDEAU<sup>(1)</sup>, who brought together the probabilistic concept of stress transmission in granular bodies (HARR<sup>(2)</sup>) and the analogous stochastically based deformation mechanism proposed by LITWINISZYN<sup>(3)</sup>, to investigate the response of particulate material to vertically applied slowly moving surface loadings.

A thorough review of existing and current literature on the mechanics of particulate media yielded the discussions on Porosity (Chapter Two), Force and Stress (Chapter Three) and Deformation (Chapter Four). As HARR and LITWINISZYN utilized an explicitly probabilistic approach, which has been adequately confirmed in the testing and coupled model of BOURDEAU, it was decided to approach the current project from the same basis. The perceived advantage of this methodology is that the response of granular materials may be characterized by a very few parameters, whereas the older stress-dilatancy and yield theories<sup>(4)(5)(6)(7)</sup> abound in extra parameters.

The testing phase was undertaken at the soil mechanics laboratory of the Swiss Federal Institute of Technology (EPFL) at Ecublens, Switzerland. A fine dry silt soil was tested under both repeated static loading and quasi-mobile loading. An array of lead shot was placed periodically throughout the soil body, and records were made of the deformation of the soil under the surface applied loading through exposure of the soil model to X-rays. Load was applied to the surface of the soil through a rigid rough

plate. In the static series of loads, the plate was repeatedly applied to the same point on the soil surface, whereas in the moving load series, the load plate was relocated contiguous to its prior position at each application. In this way, for the latter, a "quasi-moving" load mechanism was modeled. Chapter Five details the test procedures and the testing sequence. The methodology of data reduction is also outlined.

Chapter Six provides a detailed analysis and discussion of the results. The conclusion is drawn that the response of dry granular material to vertically applied moving loads is one-dimensional, and the magnitude of deformation may be estimated from an equivalent static loading. There is evidence that coefficient of lateral stress ( $\nu$ ) is not constant throughout the material under load but varies significantly with depth. The pattern of the variation of  $\nu$  with depth under the load is not inconsistent with HARR's modification of SCHMERTMANN's method of settlement analysis and lends credence to it.

Of all the data analyzed, it is evident that the primary data, that is the horizontal and vertical deformations, are the best behaved. The magnitudes of the observed vertical deformations are generally an order of magnitude greater than the observed horizontal deformations. The secondary, or derived, parameters discussed, i.e., the horizontal vertical strains, are less informative and somewhat less well behaved, while the tertiary parameters (volumetric strain and porosity) are considered only qualitatively since error propagation from the raw deformation data may be considerable.

## Chapter One

### INTRODUCTION

#### Particulate Media

The classic continuum of mechanics is distinguished by the absence of voids and hence complete continuity of matter is assumed from point to point within the body under consideration. The form, or shape of the body is defined by the geometric characteristics of its boundary. Inclusions or voids may be treated explicitly in the form of additional boundary conditions, with the necessary *a priori* knowledge of location, shape, and size. At the microscopic, or molecular, level, continua are collections of discrete particles (atoms and molecules), wherein the inter-atomic and inter-molecular forces are sufficiently strong to bind the individual elements together such that they behave as a common mass.

Granular or particulate materials, on the other hand, are generally characterized by being collections of individual particles; generally, at the visible scale. Any inter-particle attraction, in the absence of moisture, is sufficiently weak as to be sensibly neglected in engineering terms. The whole mass of a dry granular material is distinguished from a continuum by being "full of holes". The individual particles, whether spherical, cubical or irregular, cannot fit together perfectly, but rest in point-to-point contact with their neighbors. This results in

included volumes that are partially empty, or void.

Unlike continua, the voids in granular materials cannot be treated explicitly: they are so random, or chaotic, in location, shape and size as to defy rational description. Equally, the non-void, or solid component of granular materials is not spatially continuous; that is, in spite of the existence of particle-to-particle contact, there is no continuity in all directions at all points.

### Force and Stress

Classical continuum mechanics uses the stress at a point as a measure of the transmission of boundary energy within and through the continuum. This concept arises from the infinitesimal mathematics, wherein force is applied over an area, which becomes stress in the limit as the area approaches zero. Consequently, stress must be defined at a point. It is an areal parameter taken as the averaged force in the vicinity of the point. In continuum mechanics, this is generally brushed aside by invoking statistical macroscopic equivalence; however, it begins to take on very important physical meaning when applied to granular materials.

BOUSSINESQ's<sup>(8)</sup> classical solution of a homogeneous, semi-infinite elastic continuum has often been applied to granular materials. FÖPPL and FÖPPL<sup>(9)</sup>, KOEGLER and SCHEIDIG<sup>(10)</sup>, and TURNBULL, MAXWELL and AHLVIN<sup>(11)</sup> among many others have all observed and commented at length on the inability of classical mechanics to predict adequately the observed response of granular materials to loading.

In the early seventies, also concerned by the shortcomings of the classical solution, the principal investigator

(HARR<sup>(2)</sup>) developed a probabilistic approach based upon the stochastic nature of the propagation of force into a granular material. The development was predicated on the application of the methodology of a *random walk*.

### Deformation

In classical continuum mechanics, the deformation of a body under load is assumed to arise from the *induced volume changes and distortion of the solid material*: there are no voids. This is modeled mathematically by ensuring *strain compatibility*, or *continuity*, and by invoking a constitutive law, such as linear elasticity (Hooke's Law). In particulate media, there is no continuity in the classical sense as the particles are not bound to each other, so that one particle may move (translate, rotate) relative to another without necessarily affecting its neighbors. Further, deformation in granular materials results, not from the deformation of the solid material, but from relative geometric displacements between particles (BOURDEAU<sup>(1)</sup>).

### Project Objective

Building upon the work of previous studies, the objective of the current project was to gain a better understanding of the response of dry cohesionless granular material to slowly moving vertically applied surface loadings.

## Chapter Two

## POROSITY

The classic continuum of mechanics is distinguished by the absence of voids and hence complete continuity of matter is assumed from point to point within the body under consideration. The form, or shape of the body is defined by the geometric characteristics of its boundary. Inclusions or voids may be treated explicitly in the form of additional boundary conditions, with the necessary *a priori* knowledge of location, shape, and size. At the microscopic, or molecular, level, continua are collections of discrete particles (atoms and molecules), wherein the inter-atomic and inter-molecular forces are sufficiently strong to bind the individual elements together such that they behave as a common mass.

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A quantitative measure of the pore-structure of particulate materials is provided by their "POROSITY", which is defined classically as " the volume of voids in the material, as a proportion of the total material volume". Thus:

$$n_v = \frac{V_v}{V_v + V_s} = \frac{V_v}{V_T} \quad (1)$$

where  $n_v$  is the porosity (in volume terms),  
 $V_v$  is the volume of voids in the material,  
 $V_s$  is the volume of solids in the material, and  
 $V_T$  is the total volume of the material =  $V_v + V_s$

Given a unit cubic volume of particulate material, each point in the cube may be assigned coordinates,  $x, y, z$ . By sampling in the volume at a large number of random points  $R(x,y,z)$ , and noting a "hit" each time that point is in a void and a "miss" in the case of a solid, it will be noted that the ratio  $\frac{\# \text{ hits}}{\# \text{ hits} + \# \text{ misses}}$  will approach a constant. The constant value about which the sampling ratio fluctuates is referred to as the "probability of a hit"<sup>(42)</sup>. This definition of porosity may be interpreted as; porosity is the probability of finding a void in the material, or:

$$n_v = \frac{\# \text{ hits}}{\# \text{ hits} + \# \text{ misses}} = \text{Probability}[\text{void}] \quad (2)$$

From equation (1), it will be seen that, mathematically, porosity may range from zero ( $V_v = 0$  = continuum) to unity ( $V_s = 0$  = no material!). However, normal engineering soils

fall into a narrower range,  $0.2 \leq n_v \leq 0.5$ , depending upon the material gradation and degree of compaction.

HARR<sup>(2)</sup> points out that "ideal" arrangements of identical spheres yield the following theoretical porosities:

<u>Packing Arrangement</u>	<u>Theoretical Porosity, <math>n_v</math></u>
Cubic	0.4764
Orthorhombic	0.3954
Tetragonal-spheroidal	0.3019
Rhombhedral	0.2595

TERZAGHI and PECK<sup>(36)</sup> provide the following typical porosities for real soils:

<u>Soil Description</u>	<u>Typical Porosity, <math>n_v</math></u>
Loose uniform sand	0.46
Dense uniform sand	0.34
Loose graded sand	0.40
Dense graded sand	0.30
Glacial Till	0.20

SCHULTZE<sup>(40)</sup> gives a similar list of typical values including their standard deviations, without indication of degree of compaction:

<u>Soil Description</u>	<u>Typical Porosity, <math>n_v</math></u>	<u>Standard Deviation</u>
Gravelly sand	0.330	0.062
Coarse sand	0.399	0.039
Medium sand	0.404	0.041
Fine sand	0.390	0.032
Silt	0.380	0.051

The practical upper limit for porosity, assuming that the particles are in contact with at least two others (i.e., in their least stable configuration) is about 0.60; with arching in very loose materials, this might reach 0.65. The

lower limit of porosity is less well defined, and depends largely on the gradation of the particles, but it might be taken to be of the order of 0.15.

Porosity is not a material parameter, but is a state parameter. A truck load of granular base material might have a porosity of 0.40 when freshly tipped, but after compaction it may well have a porosity of 0.30. The material has not changed but its porosity has!

Given that a granular material is in its loosest state, it is in a delicate state of equilibrium. The only way that the porosity can be changed is by applying energy to the material to produce a re-arrangement of the particles. In general terms, a granular material will achieve a denser, more stable, configuration (if only locally) upon the application of energy to it, in general compliance with the principle of maximal entropy (2<sup>nd</sup> law of thermodynamics).

GRIVAS and HARR<sup>41</sup> point out that whatever the packing arrangement of the soil particles, the general measure of particle stability is given by the coordination number,  $N_c$ , which is the number of particle-to-particle contacts per particle. The greater the coordination number, the more stable the packing, in its state of local equilibrium. They point out that porosity and coordination number appear to be closely related by the relationship  $n_v \cdot N_c \approx \pi$ . (The value  $\pi$  is not a computed result, but the observed value was sufficiently close and the use of  $\pi$  lends a certain degree of technical elegance).

"Porosity at a point" has no meaning. At a point there is either a void ...or a solid. Thus porosity is a statistical property of the material, and if it is used in reference to a point location within the material, it must be understood

that it refers to a sufficient volume of the material surrounding the point to render the measure stable.

Porosity is relatively easy to measure in the laboratory, at least in a bulk sense, once it is assumed to be constant throughout the sample. In the field, porosity may be estimated from "density holes", but once again, it must be assumed that it is uniform throughout the sample.

In the course of the present research project, X-ray techniques were used (vide Chapter 5) which permitted an estimation of the porosity within discrete areas of the test material. As will be shown subsequently, it was assumed that the initial porosity was uniform throughout the material, i.e., that the material was statistically homogeneous.

Other descriptors exist that scale the state of voids within granular media. The void ratio,  $e$ , which is the ratio of the total volume of voids to the total solid volume (a transformation of the porosity). The relative density,  $D_r$ , purports to give a measure of the state of the 'densemess' of the medium, but it relies on attaining measures of the loosest and densest densities achievable in empirical laboratory tests.

The concept of porosity is central to any competent description of granular materials. There is no other parameter which so well describes the state of a soil and yet is fundamental. The understanding of porosity and of its changes is the *sine qua non* of the mechanics of particulate materials.

## Chapter Three

### FORCE and STRESS

Classical continuum mechanics uses the stress at a point as a measure of the transmission of boundary energy within and through the continuum. This concept arises from the infinitesimal mathematics, wherein force is applied over an area, which becomes stress in the limit as the area approaches zero, consequently, stress is defined at a point, it is an areal parameter taken as the averaged force in the vicinity of the point. In continuum mechanics, this is generally brushed aside by invoking statistical macroscopic equivalence; however, it begins to take on very important physical meaning when applied to granular materials.

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In 1937, POKROVSKI<sup>(17)</sup> obtained a gaussian type of distribution for stresses in a granular soil from a purely

deterministic approach. Following this work, and perhaps prompted by it, KÖGLER and SCHEIDIG<sup>(10)</sup> suggested that a probabilistic approach might be justified. The probabilistic approach was first taken up by KANDAUROV<sup>(18)</sup> and MULLER<sup>(19)</sup>, although they treated the results deterministically; not admitting, or recognizing the inherent stochastics of the system. SERGEEV<sup>(20)</sup>, in the late 60's was the first to apply direct probabilistic reasoning based upon Markovian processes. Independently, and at about the same time, SMOLTCZYK<sup>(21)</sup> was achieving the same results, but with a much less rigorous approach.

In the early seventies, HARR<sup>(2)</sup> developed a probabilistic approach based upon the stochastic nature of the propagation of force into a granular material, which also introduced the concept of the distribution of stress at a point. His development was predicated on the application of the methodology of a *random walk*. ENDLEY and PEYROT<sup>(22)</sup>, about the same time, presented the concept based on the stiffness of stress paths. In 1984, GOLDEN<sup>(23)</sup> published a thorough examination of the stochastic theory of stress propagation. While he did not add materially to the work of HARR, his more rigorous development and critical examination of the basic assumptions provided a measure of confirmation and support to the general probabilistic approach.

It should be noted that the brief historical outline presented above deals only with the concept of stress. No implication or consideration of deformation from a purely probabilistic basis was discussed, or detailed, by the noted authors. When deformation was considered, it followed from the introduction of a linearization of Hooke's law. Thus the probabilistic theory of stress propagation in granular materials has been developed in a vacuum (dare one say it...in a void).

### Contact Forces in Granular Soils

As was pointed out above, the deterministic concept of stress is a convenient artifice. The probabilistic concept will be developed below from the starting point of force, which is a fundamental physical reality and is readily measured.

The subsequent development borrows essentially from the earlier work of HARR<sup>(2)</sup>. As with the works of KANDAUROV<sup>(18)</sup>, MULLER<sup>(19)</sup> and GOLDEN<sup>(23)</sup>, it assumes that the granular medium is statistically homogeneous and isotropic and remains so throughout the loading process.

If a unit point load is applied vertically to the horizontal surface of a granular medium, then purely from consideration of static equilibrium, the applied load (force) is divided among the particles upon which the loaded particle rests. This process is repeated by those supporting particles to the next "layer" of particles. The precise determination of these reactions, their magnitudes and directions, is impossible. However, given the homogeneity of the material, and invoking the principle of maximum entropy, the least biased division is 50:50. This may be represented schematically in two dimensions as shown in Figures 1 and 2.

Mathematically, these (Figures 1 and 2) may be represented by the recursion equation:

$$F_z(x, z + \Delta z) = p \cdot F_z(x - \Delta x, z) + q \cdot F_z(x + \Delta x, z) \quad (3)$$

where  $p + q = 1$ , and  $p, q$  represent the probabilities of the force dividing to the "left" and to the "right" respectively, and  $F_z(\cdot)$  denotes the contact force in the  $z$ -direction, acting on the particle at the noted coordinate point  $(\cdot)$

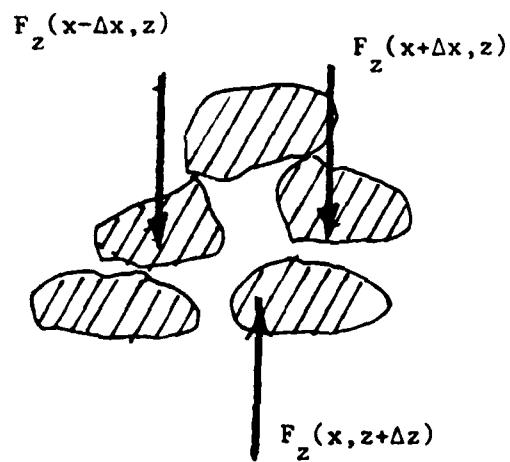


Figure 1 Resolution of particle forces in a granular medium

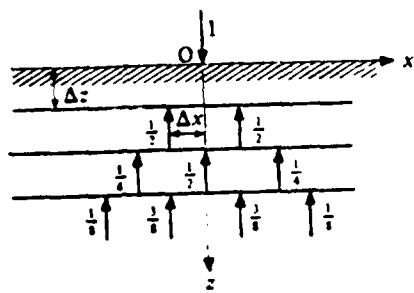


Figure 2 Probabilistic propagation of particle force in an isotropic granular medium

When  $p = q = 1/2$ , then equation (3) reduces to:

$$F_z(x, z + \Delta z) = \frac{1}{2} \cdot F_z(x - \Delta x, z) + \frac{1}{2} \cdot F_z(x + \Delta x, z) \quad (4)$$

As noted equations (3) and (4) are recurrence equations, the second of which may be transformed into the partial differential equation:

$$\frac{\partial F_z(x, z)}{\partial z} = \frac{1}{2} \cdot \frac{\partial^2}{\partial x^2} \left( D^* \cdot F_z(x, z) \right) \quad (5)$$

where:  $D^* = \lim_{\substack{\Delta x \rightarrow 0 \\ \Delta z \rightarrow 0}} \left( \frac{\Delta x^2}{\Delta z} \right)$

This is the classical Fokker-Planck equation.

Further, it may be stated that, if  $p = q$ , then the material is isotropic, but if  $p \neq q$ , the material exhibits a degree of anisotropy. GOLDEN showed that the case where  $p \neq q$  may also be evidenced in an isotropic material when the imposed force has a horizontal component.

HARR<sup>(2)</sup>, GOLDEN<sup>(23)</sup> and BOURDEAU<sup>(1)</sup> further assumed that the diffusion parameter,  $D^*$ , is a function of  $z$  only, which then removes it from the brackets under the differential with respect to  $x$ . In this case:

$$\frac{\partial F_z(x, z)}{\partial z} = \frac{D^*}{2} \cdot \frac{\partial^2}{\partial x^2} \left( F_z(x, z) \right) \quad (6)$$

where  $D^*$  is a function of  $z$  only, or  $D^*(z)$ .

Equation (6) forms the basis of the works of KANDAUROV<sup>(18)</sup>, MULLER<sup>(19)</sup>, HARR<sup>(2)</sup>, BOURDEAU<sup>(1)</sup> and GOLDEN<sup>(23)</sup>; and has the solution:

$$\bar{F}_z(x, z) = \frac{P}{\sqrt{2\pi D^* z}} \cdot e^{-\frac{x^2}{2D^* z}} \quad (7)$$

The notation  $\bar{F}_z(x, z)$  denotes the expectation of the force at that point.

The physical meaning of  $D^*$  may be understood from the observation that  $D^* z$  is the equivalent of the variance of the gaussian distribution represented by Eq. 7, and as such is a measure of the spread of the distribution of force on a horizontal plane. It is a property of the material and must necessarily be positive.

In order that tensile forces do not occur (a basic assumption in dry particulate materials) and also that the force component  $\bar{F}_z(x, z)$  tends to zero at large  $z$ , it is necessary that the product  $D^* z$  approaches zero as  $z$  increases. GOLDEN<sup>(23)</sup> shows that  $D^*$  is a power function of  $z$ , and stated that without further knowledge, the linear function (i.e., Harr's) would lead to a dimensionally balanced exponent in equation 7. As pointed out by HARR<sup>(2)</sup> and GOLDEN<sup>(23)</sup>, the simplest choice consistent with these requirements is to make:

$$D^* = \nu z \quad (7)$$

for all  $z$ , where  $\nu$  is a constant, which Harr called the "coefficient of lateral stress". Note: while Harr originally developed this concept from a consideration of stress, the development here is slightly different, based as it is on force rather than stress.

HARR<sup>(2)</sup> showed that the expected horizontal component of the force is:

$$\bar{F}_x(x, z) = v \cdot \bar{F}_z(x, z) + v^2 z^2 \cdot \frac{\partial^2 \bar{F}_z(x, z)}{\partial x^2} \quad (9)$$

Assuming that the second term on the right-hand side is small in comparison with the first term; say, for example, if  $\bar{F}_z(x, z)$  is a linear function of  $x$ , then:

$$v = \frac{\bar{F}_x(x, z)}{\bar{F}_z(x, z)} \quad (10)$$

He also pointed out the analogy between this definition of  $v$  and that of the coefficient of lateral earth pressure of classical soil mechanics,  $K$ :

$$K = \frac{\sigma_H}{\sigma_V} \quad (11)$$

For natural soils under homogeneous overburden conditions, the coefficient of lateral stress,  $v$ , may then be approximated by  $K_0$ , the coefficient of earth pressure at rest, such that:

$$D^* = K_0 \cdot z \quad (12)$$

Using the same analogy, HARR<sup>(2)</sup> proposed that  $v$  be bounded by active and passive analogues. He proposed using the REIMBERT and REIMBERT<sup>(24)</sup> inequality:

$$\left( \frac{45 - \beta/2}{45 + \beta/2} \right)^2 \leq v \leq \left( \frac{45 + \beta/2}{45 - \beta/2} \right) \quad (12)$$

where  $\beta$  is the angle of repose of the material in their formulation.

HARR<sup>(2)</sup> and GOLDEN<sup>(23)</sup>, when discussing  $\nu$ , both state "...in a homogeneous medium...", while BOURDEAU<sup>(1)</sup> states "... This important simplification is the same as neglecting the effects of structural modifications induced by loading on the way that stresses are distributed through the body at equilibrium" (Author's translation). While this subject is the topic of a later chapter, the basis of this remark stems from the belief that local volumetric deformations are directly related to changes in the local stress field. As the force field represented by Eq. (7) is obviously non-uniform in its domain, so also, the porosity of the body after loading would no longer be expected to be statistically homogeneous. It is advisable to state its implications at this point. Making the assumption that local volume changes within the body occur as the result of changes in the local forces, and that deformation of the individual particles is very small, if not negligible, and therefore that deformation proceeds as a result of changes in porosity, (i.e., local changes in the geometric arrangements of the particles); the state of stress at a point in the body should depends upon the state of the material. that is, the local stress in a dense material (low porosity) will be different from that in a loose material (high porosity), all other things being equal.

In the previous paragraphs it was stated that deformation proceeds from changes in stress, and yet, stress has not yet been defined within the present context. HARR<sup>(2)</sup> and GOLDEN<sup>(23)</sup> both treated Eq. 7 as a stress relationship; in fact HARR<sup>(2)</sup> used the notation,  $\bar{s}_z$ , for the expected value of stress, so that:

$$\bar{s}_z(x, z) = \frac{P}{\sqrt{2\pi D^* z}} \cdot e^{-\frac{x^2}{2D^* z}} \quad (14)$$

Another derivation of stress distribution will be presented that takes the material state into account more explicitly.

### Stress

Consider a small volume of a granular medium surrounding the point  $(x, z)$ , large enough to define a competent statistical sample of particles. Within that sample there are voids, which cannot sustain force, and solids which accomodate to all induced forces. The forces acting on each particle are not equal, but are distributed about a vertical component of a mean value,  $\bar{F}_z(x, z)$ . Thus, we may write:

$$\text{Prob}[\text{void}] \cdot \left[ \begin{array}{l} \text{vertical} \\ \text{force in void} \end{array} \right] + \text{Prob}[\text{solid}] \cdot \left[ \begin{array}{l} \text{vertical} \\ \text{force on particle} \end{array} \right] \\ = \left[ \begin{array}{l} \text{vertical} \\ \text{forces averaged throughout the volume} \end{array} \right]$$

Similar relationships may be written for forces in any geometric orientation.

As was shown in Chapter Two, the probability of finding a void in a particulate material is the porosity,  $n_v$ . Thus, as there are no forces in the voids of a dry material:

$$n_v \cdot 0 + (1 - n_v) \cdot \bar{F}_z(x, z) = \bar{s}_z(x, z) \quad (15)$$

where,  $\bar{s}_z(x, z)$  is the expected value of the expectation of the vertical forces in the volume surrounding the point  $(x, z)$ .

Now from equation 15, it may be seen that:

$$\bar{s}_z(x, z) = (1 - n_v) \cdot \bar{F}_z(x, z) \quad (16)$$

which shows that  $\bar{S}_z(x,z)$  is a unique value for each distribution of force and the inherent porosity of the material.

It is proposed that the variable  $\bar{S}_z(x,z)$  in equation 16 be called the expected value of local stress at the point  $(x,z)$ . It is seen then that it is a function of the local material state (porosity), whereas, local forces are not. A material body under loading becomes statistically nonhomogeneous; but while local contact forces may not change, the state of stress will reflect changes in porosity. Further, during settlement (or consolidation) under a constant applied force, the local stresses and porosities must exhibit the relationship:

$$\frac{S_z(x,z)}{(1 - n_v)} = \text{Constant} \quad (17)$$

So, for example, if a material exhibited a local stress of  $\bar{S}_z(x,z)$  at a porosity of 0.4, and after some time the porosity had changed to 0.35 under the same loading conditions (force field), the local stress would change to  $1.083 \cdot \bar{S}_z$ , an increase of 8%; again without any increase in local forces. This duality of local porosity and local stress is one of the central ideas of the present study.

## Chapter Four

## DEFORMATION

In classical continuum mechanics, the deformation of a body under load is assumed to arise from the induced volume changes and distortion of the solid material: there are no voids. This is modeled mathematically by ensuring strain compatibility, or continuity, and by invoking a constitutive law, such as linear elasticity (Hooke's Law). In particulate media, there is no continuity in the classical sense as the particles are not bound to each other, so that one particle may move (translate, rotate) relative to another without necessarily affecting its neighbors. Further, deformation in granular materials results, not from the deformation of the solid material, but from relative geometric displacements between particles (BOURDEAU<sup>(1)</sup>, MARSAL<sup>(25)</sup>).

The behavioral differences between particulate and continua media is fundamental. In the latter case, the deformation of an elementary volume (3D) or area (2D) must be compatible with those elements immediately surrounding it, regardless of the adopted constitutive model. This is a result of the requirements of continuity, or compatibility. In elastic models (favored for superposition effects), deformation follows immediately upon application of load, and is immediately reversible upon the removal of that load. In other models (plastic, visco-elastic etc.), this is not a requirement, but then the benefits of superposition are lost, and in general, there is an increase in the number of

parameters required to characterize the material behavior.

The stochastically-based models of deformation in particulate media proposed by MARSAL<sup>(25)</sup>, LITWINISZYN<sup>(3)</sup> and BOURDEAU<sup>(1)</sup> do not require 'continuity'; they intrinsically incorporate transience, or 'time effects' and are parsimonious in terms of parameters. It may be that these models are overly simplistic, but their proven advantage lies in the observed immediate improvement in predictions of steady-state deformations.

LITWINISZYN<sup>(3)</sup> (the doyen of the Polish school) developed a comprehensive stochastic model for the deformation of particulate media resulting from volume changes within the media (e.g., the deformation resulting from the collapse of mining galleries): upon the collapse of a gallery, the surrounding rock "falls" into the void, thereby exchanging position with an equivalent volume of "space", or "void". It is this progressive mechanism which he hypothesized that causes the ultimate surface deformation and accounts for the transient effect. BOGDANOFF and SWEET<sup>(26)</sup> extended this work. MARSAL<sup>(25)</sup> made the connection with stressed media (rather than 'collapsed' material), where the initiating internal volume change resulted from local changes in the stress field. BOURDEAU<sup>(1)</sup> showed that while MARSAL's<sup>(25)</sup> stochastic model was organically correct, the basic assumption of particle 'migration' could not be supported: he proposed the coupling of HARR's stress propagation model with LITWINISZYN's<sup>(3)</sup> void 'migration' deformation model. BOURDEAU<sup>(1)</sup> treated the case of static surface loads applied to a portion of the surface of a soil body: transience was considered only as component of the steady-state deformation.

### Deformation of Granular Media

There are three primary mechanisms of deformation in granular media<sup>(25)</sup>:

- i) Elastic deformation of the solid particles,
- ii) Particle crushing, and
- iii) Particle realignment and relocation.

#### Elastic Deformation

The elastic modulus (E) and Poisson's ratio ( $\mu$ ) of a silica sand sphere (grain) are typically 6,000,000 lb/in<sup>2</sup> and 0.15 respectively<sup>(28)</sup>. Uniaxial deformation under a unit diametrically applied load will be of the order of 0.0001 inches in a particle 0.04 inches in diameter, or an overall strain of 0.35%<sup>(29)</sup>. Observed deformations of assemblages of granular materials generally exceed these values by orders of magnitude. Undoubtedly, in a loaded granular medium, there will be a component of elastic particle deformation, which will be recoverable upon removal of the load. The magnitude of this component has been found to be negligible.

#### Particle Crushing

The expected values of the crushing strengths of typical granular materials (in civil engineering applications) are of the order of 30,000 lb/in<sup>2</sup> (2160 ton/ft<sup>2</sup>)<sup>(27)</sup>. Once again, this is far in excess of the expected value of stresses induced within a loaded granular mass. While expected values of stresses in granular bodies may be relatively low, point contacts between particles can engender extremely high local stresses sufficient to cause crushing. This mechanism is progressive until the contacts have degraded to such an extent that the area of contact has

increased sufficiently to reduce the local stresses below critical levels. In some rock and granular fills, where the quality of the stone is either variable or poorly controlled, there may well be some individual particles which undergo crushing, however, in general practice, this should be relatively rare<sup>(27)</sup>. While there will be some deformation resulting from particle crushing, the incidence of significant crushing is negligible.

#### Particle realignment and relocation.

The deformation under load of particulate media is based upon the premise that in a state of equilibrium, or steady-state, the forces acting on each particle are in balance. With increases in the imposed loads, local changes occur in both the magnitude and direction of the forces acting on the particles. The departure from static equilibrium, may render particles unstable, in which case they may move to a more stable configurations. If the particles in a local volume remain in undisturbed equilibrium, there is no change in the volumetric distribution of solids and voids in that region. However, if, in order to acquire a state of enhanced equilibrium, the particle slips or rotates, then there may result a local re-arrangement of the particles. As BOURDEAU<sup>(1)</sup> and others have shown, this leads to a redistribution of the local solid density. If the motion leads to a consequent local densification, the "excess" voids successively exchange positions with those of particles located above. That is to say, the "voids" travel upward to the surface in much the same way as gas bubbles in a liquid.

BOURDEAU<sup>(1)</sup> pointed out that if a single particle is removed from a stable particulate arrangement the newly created void will (can) be replaced by a solid particle

(thus increasing the local density); but, thereafter on its random flight through the material, the exchanged "bubble" of air has no further densifying effect, since the 'before' and 'after' situations are essentially undisturbed. Figure 3 schematically demonstrates this point. Due to a change in local stresses, the void "particle" a, is replaced by solid particle b, thus increasing the local density and stability. The new void b, is in turn displaced by solid particle c, and so on. After all movement has ceased, the local density in frame A has increased due to the capture of particle b to a, while the local density in frame B remains unchanged, since its mass balance remains constant, i.e., it has both gained a particle (e) and lost a particle (c).

The following is based on the work of LITWINISZYN<sup>(3)</sup>, and provides a steady-state model of the effect of void migration. Denoting the volume of a void displaced at point (x, z) within the body as  $w_0$ , the recurrence relationship for the passage of the void to the surface may be given as:

$$w_0(x, z+\Delta z) = \frac{1}{2} \cdot \left( w_0(x-\Delta x, z) + w_0(x+\Delta x, z) \right) \quad (18)$$

$$w_0(x, z+\Delta z) - w_0(x, z) = \frac{1}{2} \cdot \left( w_0(x-\Delta x, z) - 2w_0(x, z) + w_0(x+\Delta x, z) \right) \quad (19)$$

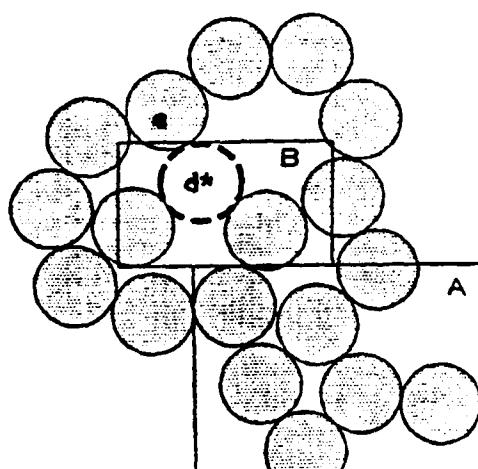
$$\frac{\partial w_0(x, z)}{\partial z} = \frac{1}{2} \cdot \Omega \cdot \frac{\partial^2 w_0(x, z)}{\partial x^2} \quad (20)$$

where

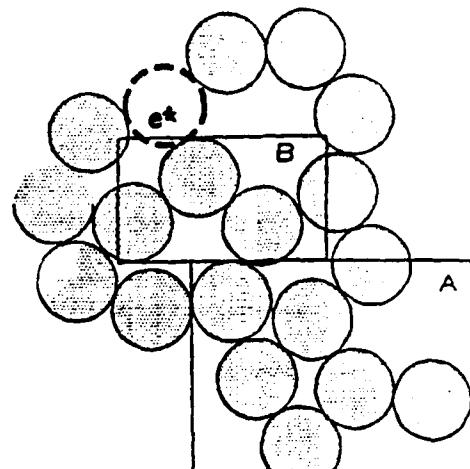
$$\Omega = \text{Lim} \left( \frac{\Delta x^2}{\Delta z} \right)$$

Again, as in equation (14) the solution may be written as:

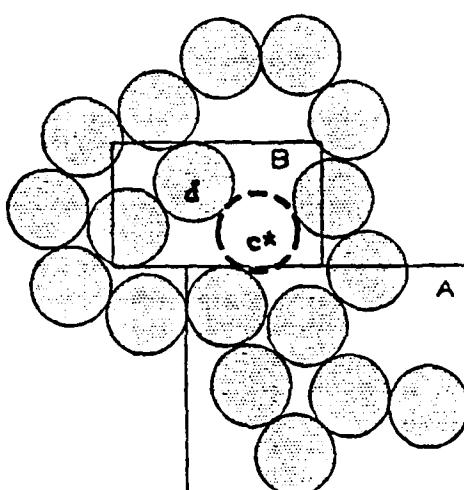
$$w_0(x, z) = \frac{w_0}{\sqrt{2\pi\Omega z}} \cdot e^{-\frac{x^2}{2\Omega z}} \quad (21)$$



(IV)

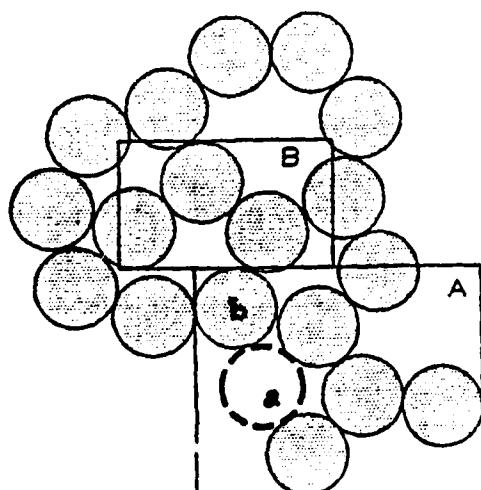


(V)

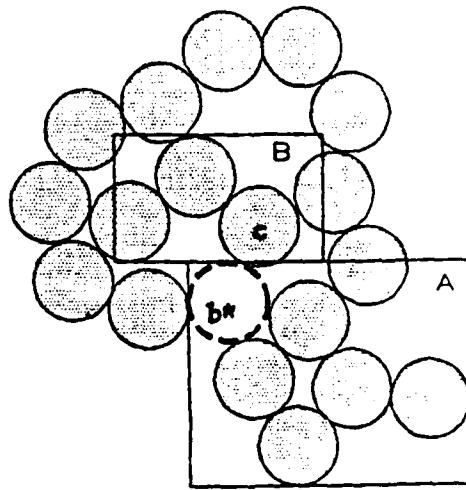


"particles = lower case  
and voids

(III)



(1)



(11)

Figure 3 Local effects of void-particle interchange

This relationship provides a measure of the location of the void in the body but not its size. The initiating void volume,  $w_0$ , is a function of particle sizes, and the skeletal arrangement of the grains at the point of origination of the void. To make the orientation of the "z" variable in the stress and void propagation relationships compatible, the above equation may be rewritten:

$$w_0(x, z) = \frac{w_0}{\sqrt{2\pi\Omega(h-z)}} \cdot e^{-\frac{x^2}{2\Omega(h-z)}} \quad (22)$$

where  $h$  is the depth below the surface of the originating solid-void exchange.

BOURDEAU<sup>(1)</sup> defines the change in local volume from a local area within the body as:

$$\Delta V = \frac{\Delta\sigma}{\beta} \cdot V \quad (23)$$

where  $\Delta\sigma$  is the local change in the first stress invariant,

$\Delta V$  is the local volume change,

$V$  is the initial volume,

$\beta$  is a "compression modulus"

In this case, it will be seen that the local volume change  $\Delta V$ , is the sum of the individual voids lost to the volume, or:

$$\Delta V = \sum_i w_0^i(x, z)$$

The local volumetric strain may be written as:

$$\Delta V/V = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 \quad (24)$$

where the  $\varepsilon_i$  are mutually orthogonal normal strains.

Pleading linear elasticity with respect to equation (24), it can be demonstrated that the parameter  $\beta$  in equation (23) may be expressed in terms of linear elastic constants,  $E$  and  $\mu$ :

For 2-D (plane strain):

$$\varepsilon_x + \varepsilon_z = \frac{1-\mu^2}{E} \cdot \left( \sigma_x + \sigma_z - \frac{\mu}{1-\mu} \cdot (\sigma_x + \sigma_z) \right) \quad (25a)$$

or

$$\varepsilon_x + \varepsilon_z = \frac{1+\mu}{E} \cdot (1-2\mu) \cdot (\sigma_x + \sigma_z) = \frac{1}{\beta} \cdot (\sigma_I + \sigma_{II}) \quad (25b)$$

thus

$$\beta = \frac{E}{(1+\mu)(1-2\mu)} \quad (25c)$$

For 3-D

$$\varepsilon_x + \varepsilon_y + \varepsilon_z = \frac{1}{E} \cdot (\sigma_x + \sigma_y + \sigma_z - 2\mu(\sigma_x + \sigma_y + \sigma_z)) \quad (26a)$$

or

$$\varepsilon_x + \varepsilon_y + \varepsilon_z = \frac{1-2\mu}{E} \cdot (\sigma_x + \sigma_y + \sigma_z) = \frac{1}{\beta} \cdot (\sigma_I + \sigma_{II} + \sigma_{III}) \quad (26b)$$

thus

$$\beta = \frac{E}{(1-2\mu)} \quad (26c)$$

It is arguable that these relationships hold true for a granular material, but in any case, they do strike an interesting analogy.

BOURDEAU<sup>(1)</sup>, in using his relationship, implicitly accepted deformation as a continuous reversible and linear function

of stress. However, upon the removal of load from such a body, deformation is not fully recovered, (consider, for example, the permanency of footsteps in sand). Upon the replacement of the load, deformation is essentially linear up to the previous maximum magnitude of load; thereafter, deformation is non-linear and transient. (Analogous to precompression and virgin compression in the consolidation of clays). Although the deformation is linear in the "elastic" or pre-loaded range when the surface is re-loaded, this holds true only when the load is re-applied at its original location on the surface. When the load is re-applied at a different surface location, the induced stress state at a point in the body may indeed not exceed the previous magnitude of the trace ( $\sigma_1 + \sigma_{11} + \sigma_{111}$ ); but the principal directions may well have changed orientation. This possibility is not accounted for in equation (23).

The exhibited "elastic" portion of the load - reload response may be explained as follows. Consider the state of a statistically valid volume around some point  $(x, z)$  in a granular body under a given steady-state stress regime wherein the particles are all in static equilibrium. This equilibrium is not necessarily critical. If the stress state changes, the equilibrium between the particles may be disturbed and a permanent, or irreversible deformation will occur. However, it is wholly possible that the equilibrium state prior to loading is sufficiently robust under the increment of load so that the particles retain their relative positions, in which case the applied energy is absorbed by elastic deformation of the particles.

Unlike the concept of stress, deformations are directly measurable. The experimental phase of the present project will make specific use of this advantage to monitor the internal deformations of granular bodies during loading.

In summary, HARR<sup>(2)</sup> developed the concept of stress in granular materials without consideration of deformation and LITWINISZYN<sup>(3)</sup> theorized the mechanism of deformation without consideration of stress. BOURDEAU<sup>(1)</sup> was the first to bring together the two disparate hypotheses within a probabilistic (rather than from a deterministic) framework.

## Chapter Five

### EXPERIMENT

The experimental phase of this research project fell into two clearly definable parts: the response to repeated static loadings and to incremental moving loadings. Consequently, the initial experiments were set up to identify and quantify material parameters under static loading tests. Further, it was reasoned, if the static case could be fully modeled mathematically the extension to the moving load case would be greatly facilitated.

The laboratory facilities of the École Polytechnique Fédérale de Lausanne (EPFL), at Ecublens in Switzerland were kindly made available to the authors. The first step was to construct of physical model (Figure 4) to contain soil to be tested and to permit the implantation of a regular array of lead shot. Loadings would be introduced at various positions on the soil surface through a rigid plate. Changes in the locations of the shot induced by the surface loadings would be monitored by remote sensing (X-rays). This test regime provided the experimental data for the project.

#### Test Model

The test model was a glass-sided container with internal dimensions, 80 cm long by 68 cm deep by 19 cm thick. The glass sides were 1.5 cm thick (Figure 4).

Loads were applied to the surface of the material through a

15 cm by 19 cm rigid plate. A sheet of sandpaper was glued to the base of the plate to provide a "rough", frictional interface between the load and the soil. The load was varied by a hydraulic pump. The rigidity of the load support frame was sufficient to ensure that the load was applied vertically to the soil. The loading plate could be moved and locked into position so that adjacent loaded areas were contiguous.

Two dial gauges were employed to measure vertical displacements, one on each side (left and right) of the load plate. These allowed a check to be made on the "tilt" of the loading plate, and permitted the operator to judge whether deformations had achieved a constant value. The criterion used to gauge final settlement was that used by BOURDEAU<sup>(1)</sup>, i.e., Swiss Standard SN 670'327, which set a settlement rate of less than 0.02 mm/min as a practical end point.

Measurements of spatial deformations were achieved by placing an array of lead shot throughout the material in the model and registering their locations at given times during the tests by exposing the model to X-rays, the positions of the lead shot being recorded on photographic film (Fig. 5).

The details of the X-ray emitter are:

Type:	Phillips MG150
Tube:	MÖD 151/Be
Aperture:	3 x 3 mm
Focus:	60 cm
Film:	Kodak Industrex CX
Intensity:	28 mA
Power:	120 kV
Exposure:	6-12 min

Table 1.  
Technical Data

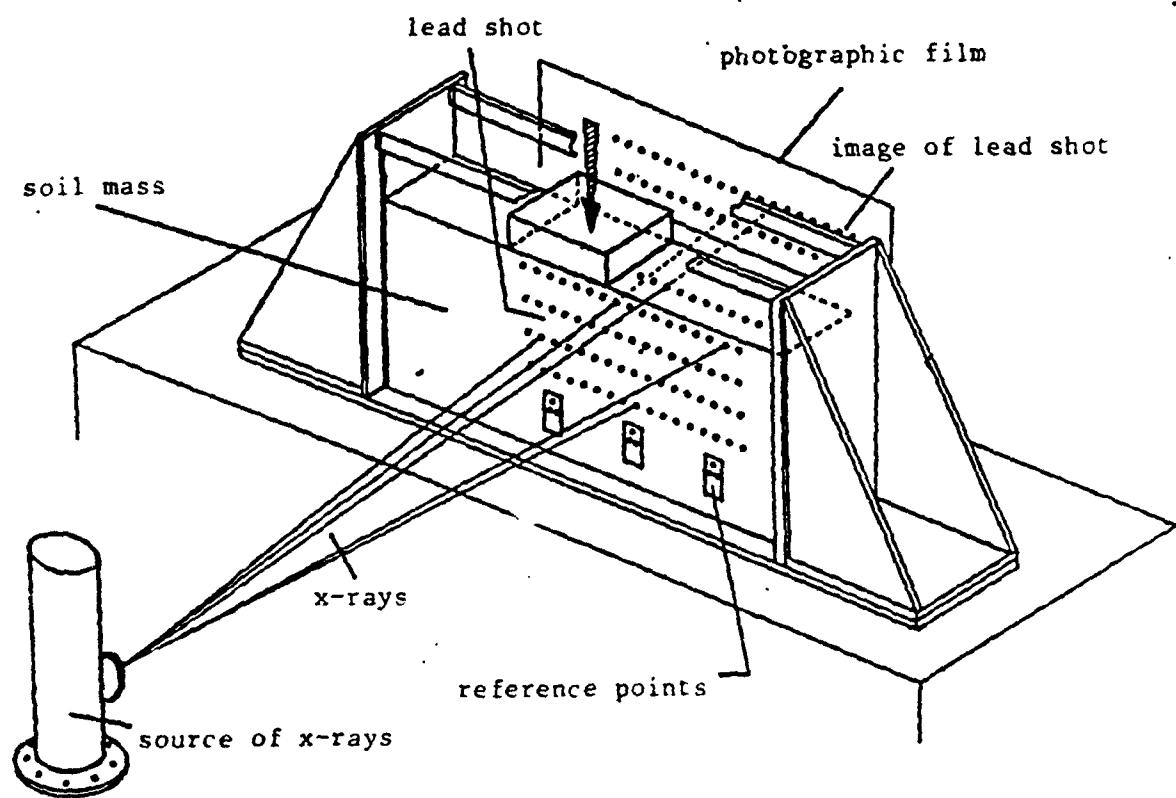


Figure 4 Schematic view of test mode and x-ray

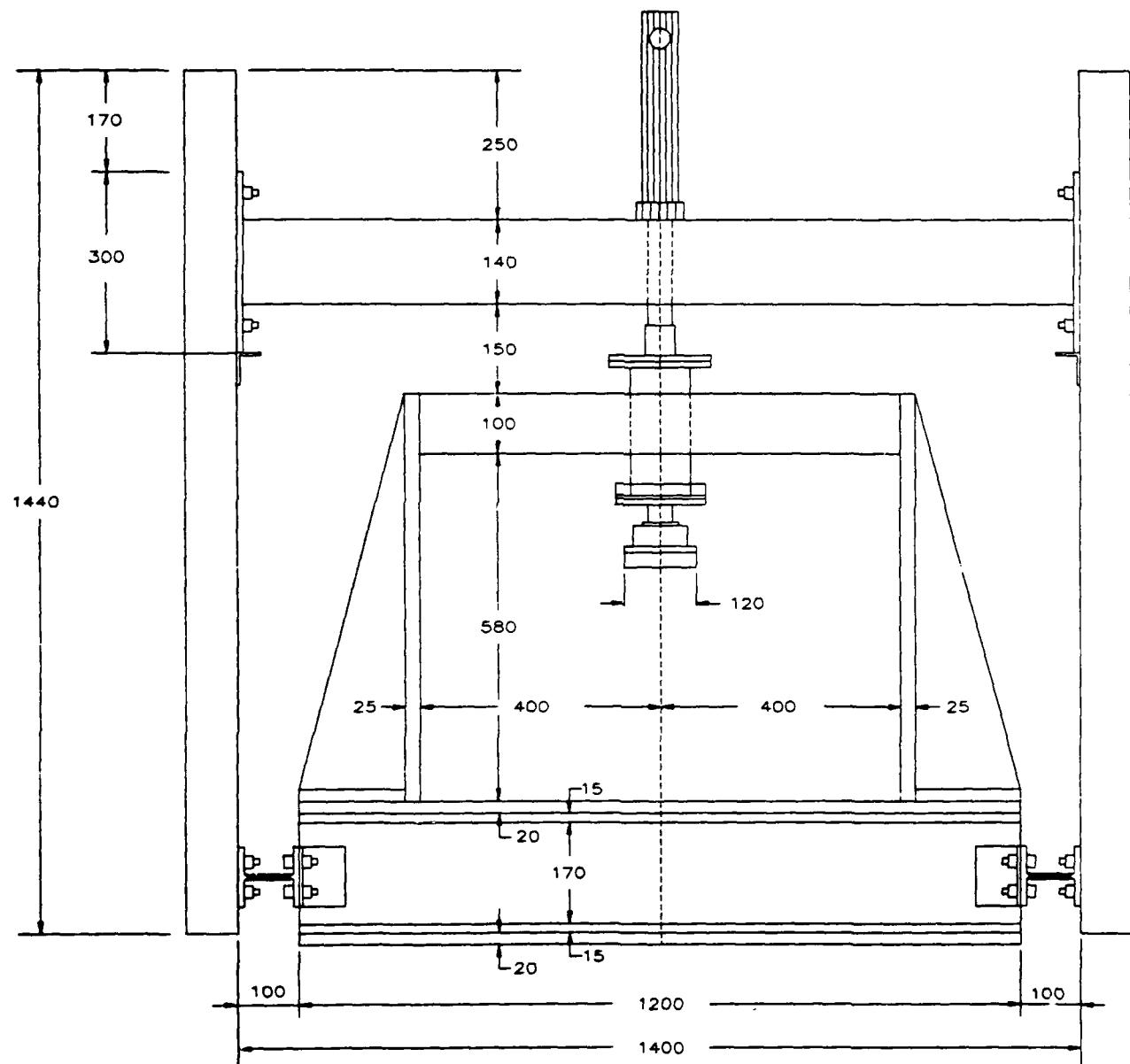


Figure 5 Front view of test model

### Test Material

The soil used in the tests was a well-graded silt from the Bolley-Orjulaz quarry, Switzerland, from which the gravel fraction had been scalped. All standard laboratory tests on the material confirmed it as being an essentially cohesionless when dry (Tables 2 - 4)<sup>(1)</sup>.

Gradation		Consistency	
$d_{10}$ (mm)	<0.001	Liquid Limit	32
$d_{30}$ (mm)	0.0045	Plastic Limit	19.4
$d_{60}$ (mm)	0.02	Plasticity Index	12.6
$d_{60}/d_{10}$	>20		
$(d_{30}^2)/d_{10}/d_{60}$	>1	USCS Classification	CL
$\bar{d}$ (mm)	0.04		
$s_d$ (mm)	0.10	Composition:	
$V_d$	250	Carbonate	60%
$\gamma_s$ (kN/m <sup>3</sup> )	27.1	Quartz	15 - 20%
		Muscovite	20 - 25%

Table 2. Test Material Particle Characteristics  
(after BOURDEAU)

Direct Shear			
moist, unconsolidated	$\phi_{pic} = \phi_{res} = 29$	$c = 0$	
saturated, consolidated	$\phi_{pic} = \phi_{res} = 29.5$	$c = 0$	
Simple Shear			
saturated, consolidated	$\phi_{pic} = \phi_{res} = 30.5$	$c = 0$	
Triaxial			
saturated, consolidated (CD)	$\phi_{pic} = \phi_{res} = 33$	$c = 0$	
saturated, consolidated (CU)	$\phi_{pic} = \phi_{res} = 33^\circ$	$c = 0$	
saturated, consolidated (CU)	$\phi_{pic} = \phi_{res} = 34^\circ$	$c = 0$	

Table 3. Test Material: Shear test Results (after BOURDEAU)

Maxima		Minima	
$\gamma_d$ (kN/m <sup>3</sup> )	17.85	$\gamma_d$ (kN/m <sup>3</sup> )	11.92
$e_{max}$	1.27	$e_{min}$	0.52
$n_{max}$	56.10	$n_{min}$	34.20

Table 4. Test Material: Compaction Limits (after BOURDEAU)

Test materials were in a dry condition for all tests, having been dried previously in an oven to constant weight. Moisture content determinations were taken from each test and indicated moisture contents less than 1%.

Soil to be tested was placed in the test model in approximately one inch layers. The material was rained gently from a scoop from a height of approximately one inch. The surface of each layer was carefully leveled to minimize disturbance. After each layer was placed, lead shot (average diameter 2.5 mm) were placed into position in the center plane of the model from a device which spaced them at approximately 1/2 inch spacing, while ensuring that they lay in the central vertical plane of the model.

When the model was full (approx 40 cm deep) its final dimensions were recorded as was the weight of soil. From this, its bulk density was computed. Knowing the solid density of the silt, the bulk porosity was then computed and used as the initial reference porosity.

Total Material Weight	$W_t$
Total Material Volume	$V_t$
Bulk Density, $\gamma_B$	$W_t/V_t$
Solid Density, $\gamma_s$	$\gamma_s$
Bulk Porosity, $n_v$	$\frac{100 \cdot (\gamma_s - \gamma_B)}{\gamma_B \cdot (\gamma_s - 1)}$

All soil porosities computed during the course of the testing were based on differences from the initial porosity, which was assumed to be uniform throughout the model.

#### Deformation Registration

After loading, the movement of the lead shot was recorded on photographic film after exposure to X-rays. In order to provide fiduciary reference, two series of lead shot (24 shot total) were glued to the glass walls of the model: these, being at a fixed position for every subsequent exposure, provided a reference basis.

The first exposure of each model test was made with the model empty and with the photographic film held vertically in the plane of the shot (i.e., midway between the glass walls). This initial exposure provided the coordinates to which the reference shot images would be reduced for each subsequent exposure throughout the remainder of the tests.

The next exposure was taken with the model filled, but unloaded i.e., under virgin conditions. In this state and subsequent exposures, the photographic film was held vertically outside the model. Subsequent exposures were made while the material was under load, or after removal of loading. Geometric rectification of these exposures provided the coordinates of the lead shots at each step of the experiment, reduced to the physical plane of the shot within the model.

The coordinates of the images of the lead shot were abstracted from the developed negatives on a table digitizer. The coordinates were rectified by the eight-parameter method<sup>(30)</sup> (i.e., scale, rotation and

translation) and reduced to the plane of the shot in the model. The transformation relationships are given by:

$$y'' = \frac{a_0 + a_1 \cdot x' + a_2 \cdot y'}{1 + b_1 \cdot x' + b_2 \cdot y'} \quad x'' = \frac{a_3 + a_4 \cdot x' + a_5 \cdot y'}{1 + b_1 \cdot x' + b_2 \cdot y'} \quad (25)$$

where the eight parameters,  $a_{1-5}$  and  $b_{1-2}$ , are found by least squares regression using the coordinates of the images of the 24 fixed reference shot in the first exposure  $(x'', y'')$ , and the corresponding 24 image coordinates on each successive exposure  $(x', y')$ . The resulting reduced coordinates were found to be more satisfactory than those produced by the simpler five-parameter transformation initially used by BOURDEAU<sup>(1)</sup>.

Exposure times of the photographic plates to the X-rays were found to be variable as the density and definition of the negative material was sensitive to the maturity of the developing and fixative solutions. When fresh, exposures times could be as little as six minutes, while an exhausted developer required an exposure of as much as 15 minutes to provide satisfactory image contrast.

The required exposure duration was a limiting factor in attempting to capture transient effects. However, little could be done to improve the system. No other available system could provide the internal metric equivalent of an X-ray "movie".

#### Test Series

The first series of tests (October - November 1987) were inconclusive for a number of reasons, among these were:

- a. The image of the reference exposure was poor and the digitizing process of the lead shot images was

almost impossible to obtain with any reasonable degree of accuracy.

- b. During the course of the tests, the loading plate was accidentally dropped onto the surface of the model, seriously disturbing the material, and rendering the rest of the first series invalid.

While the first series of tests was inconclusive from a numerical data gathering point of view, the experience was valuable in the sense of providing operator familiarity and "shakedown". It resulted in a number of procedural changes which were implemented in the succeeding series that led to the successful completion of the testing program.

The second series of tests (March 1988) were again marred by difficulties; however, two successful sets of test results were obtained. The STATIC LOAD series of tests was satisfactorily conducted, while the initial MOVING LOAD series was aborted due to disturbance of the model. A second MOVING LOAD series was also satisfactorily completed.

The final series of tests (June - July 1988) was completely successful. Both STATIC LOAD and MOVING LOAD tests were conducted and the results were recorded. It is of special note that the results of the two STATIC LOAD series (March and June - July) are essentially indistinguishable. This observation was disappointing insofar as no further information was gained from the second test, but nonetheless encouraging in confirming that the results were reproducible.

### Static Load Series

In the static load series of tests, the undisturbed model was initially subjected to a central, symmetrically placed load of low magnitude ( $3.8 \text{ kN/m}^2$ , 0.55 psi). The load was maintained constant until deformation had stabilized. This took approximately three to four minutes. An X-ray exposure was then taken and the load was removed. Then another exposure was made. The load was replaced and increased ( $7.2 \text{ kN/m}^2$ , 1.04 psi) and held constant until deformation had stabilized: an exposure taken. The load was then removed and another exposure was made. This procedure was repeated for each load increment.

### Moving Load Series

In the two MOVING LOAD series, a similar regime was followed as for the static case, except that after the deformations due to the applied load was found to be constant it and was translated laterally so as to be contiguous with the previous load location. In each series there were five load locations.

This sequence of locations was intended to represent "snapshots" in time for a laterally moving load. The discretization into steps was dictated by the physical size of the footing and the bolt hole locations in the overhead beam. The time required to achieve constancy of deformation was extremely short. Much more time was required to effect the exposure. The time required to conduct the sequence of events could not be reduced with the apparatus available to the authors at the EPFL.

The reductions of the X-ray exposures for both the STATIC and MOBILE test series were labelled as follows, and are so

identified in Appendix A:

A. The STATIC series are identified by the word "STATIC" followed by the magnitude of load, e.g.,  $38.1 \text{ kN/m}^2$ , followed by the parameter shown, as:

STATIC -  $3.8 \text{ kN/m}^2$  - Horizontal Deformation

which designation describes the STATIC series exposure under a load of  $3.8 \text{ kN/m}^2$ , and the figure shows the contours of horizontal deformation.

In the STATIC series, only the following are shown in the appendix:

Horizontal Deformation,  
 Vertical Deformation,  
 Horizontal Strain (%),  
 Vertical Strain (%).

B. The MOBILE load series was identified by the word "MOBILE" followed by the code descriptor given in Table 5:

Position	Load	
	$16.2 \text{ kN/m}^2$	$35.0 \text{ kN/m}^2$
0	0.0	1.0
1	0.1	1.1
2	0.2	1.2
3	0.3	1.3
4	0.4	1.4

Table 5. Reference Sequence for MOBILE Loadings

Following this descriptor, the parameter of interest is given as in the STATIC series, but with these additions:

Volumetric Strain (%),  
 Porosity.

## Chapter Six

### DISCUSSION OF RESULTS

The results of primary concern with respect to the present study are the vertical and horizontal deformations within the body of the soil induced by the loadings applied to its surface. The derived strains and porosities are of secondary interest. The validity of conclusions drawn from these secondary results depends upon the robustness of the primary data.

#### Time effects during loading

The theories of LITWINISZYN<sup>(3)</sup> and BOURDEAU<sup>(1)</sup> are based on the transient response of granular materials to changes in boundary stresses. In the present study, the time elapsed from the application of load (to the model surface) to apparent final state was very short, generally of the order of one minute. Clearly, any transient process had ceased by the time the X-ray exposures were made. Exposure times were of the order of 6 - 10 minutes. Moreover, had there been any significant movements of the lead shot they would have been highlighted by the apparent elongation of their recorded images. None were seen on the developed films. Analyses of the results are made on the assumption that all recorded movements of the shot were "instantaneous". This brings into question the principal response mode. Significant transience implies a "consolidation" mechanism, however the evidence of the present study demonstrated that a mechanism more similar to compaction than consolidation

may predominate. Whereas the classical mechanism of consolidation is based upon the dissipation of excess pore pressures, progressively transferring the load from the pore fluid to the solid skeleton, the response of a dry cohesionless material appears to be instantaneous. That is, boundary forces in the compaction mode are transferred almost immediately to the soil skeleton. The corresponding deformations as a consequence of the relative motions of particles forming stable configurations.

#### Incrementally moving load

Whereas the loading scheme used in Switzerland was by necessity incremental in space, it cannot truly (physically) represent a moving, or rolling, load. Nonetheless, in light of the strong one-dimensional deformation response observed in the tests and to be discussed later, the representation is believed to be materially similar to that of rolling vehicles. That is; neglecting momentum effects, on the basis of the results it would be expected that the response of the soil body would be the same for either a rolling or incrementally moving load so long as the amount of energy imparted per unit volume of the soil body is the same in both cases in magnitude and application. The reported test results relate to a single pass of the loading plate over the surface of the model. However, in a repeated load passage scenario the cumulative response of the material to a rolling or an incrementally applied loading scheme must become less and less significant.

#### Vertical and horizontal deformation

##### Static Loading

It may be seen from Figs. A1 thru A24 (Appendix A) that the

vertical deformations are much more pronounced and definitive than the corresponding horizontal deformations. Significant horizontal deformations are seen to be concentrated mainly at the edges of the loaded plate. The maximum vertical deformation is approximately an order of magnitude greater than the maximum horizontal deformation. It is difficult without direct observation to determine to what extent the horizontal deformations are influenced by soil-wall friction, however it is reasonable to expect that significant wall friction would more likely influence or even mask the relatively small deformations than the vertical deformation. Consequently, care should be exercised for making any but qualitative assessments based on the reported horizontal deformations.

The "shapes" of vertical deformation curves observed in the model, are not markedly different from those predicted by other methods. Figures 7 and 8 show observed and computed normalized deformations under the center of a single strip load. The theoretical values follow from Egorov<sup>(31)</sup> (Figure 7) and Kandaurov<sup>(18)</sup> (Figure 8). Kandaurov derived a "granular" stress distribution, similar to that of Harr, and postulated a modified "Hookean" deformation, whereas Egorov's solution was obtained from pure classical linear elastic continuum mechanics.

Kandaurov's solution considered a flexible loading on a finite layer of soil of thickness  $h$ : his solution was essentially one-dimensional. He began by assuming the constitutive relationship :

$$\epsilon_z = \frac{1}{E} \cdot \sigma_z \quad (27)$$

and thus:

$$\Delta_z = \frac{1}{E} \int_{z=0}^{z=h} \sigma_z \cdot dz \quad (28)$$

Fig 7. Comparison Egorov vs Experiment

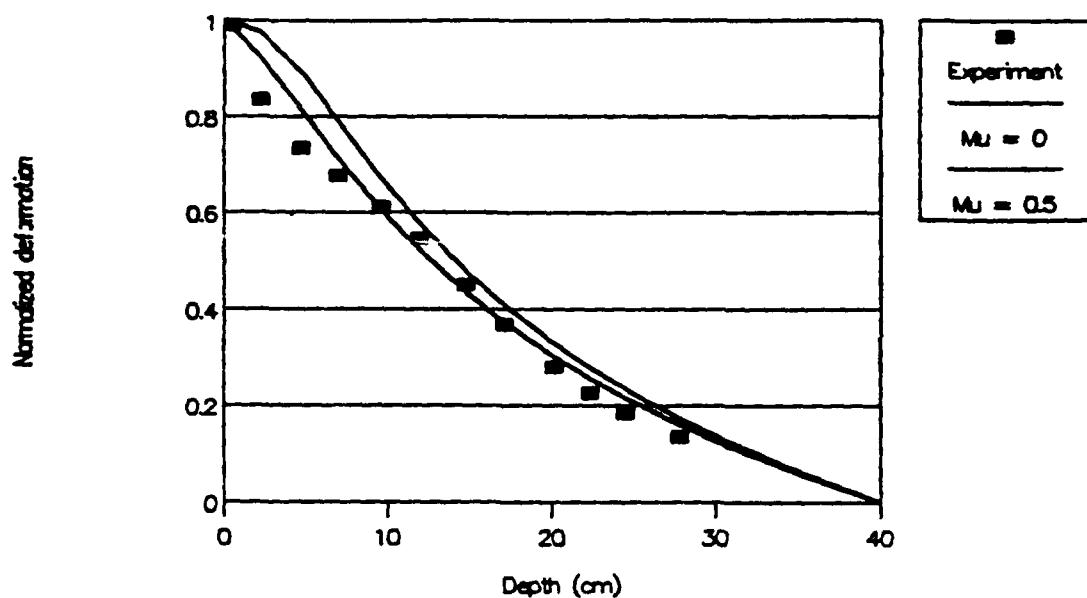
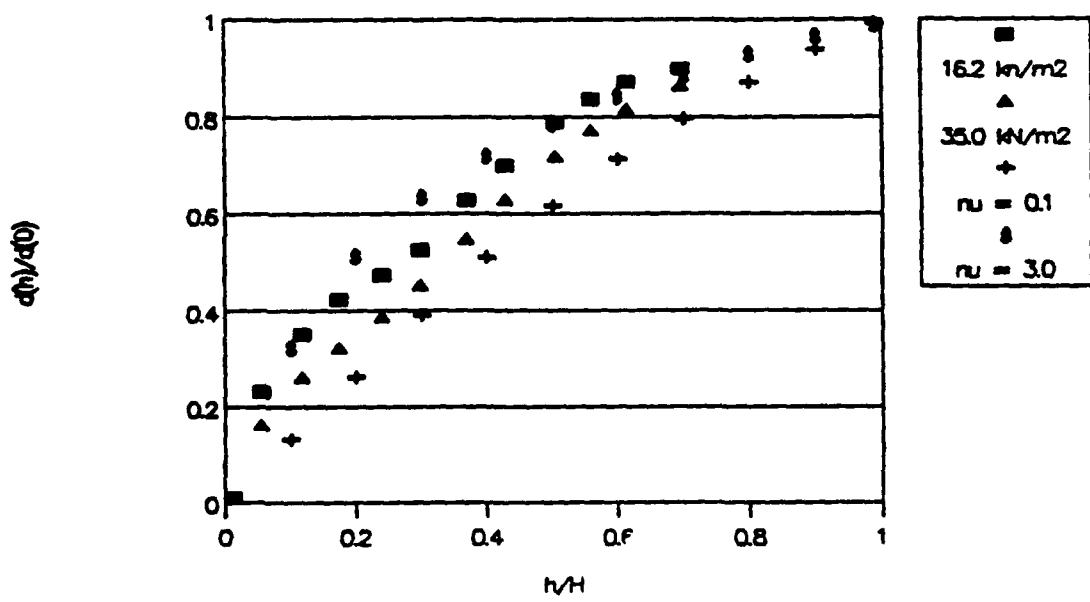


Fig 8. Comparison Kandaurov/Experiment



which under the axis of load becomes:

$$\Delta_z = \frac{p}{E} \cdot \int_0^h \Phi \left( \frac{b}{2 \cdot \sqrt{\nu}} \right) \cdot dz \quad (29)$$

where

$\Delta_z$  is the vertical deformation at depth  $z$ ,  
 $p$  is the magnitude of applied loading,  
 $E$  is Young's Modulus of the soil,  
 $b$  is the half-width of the strip load  
 $\nu$  is the Coefficient of Lateral Stress

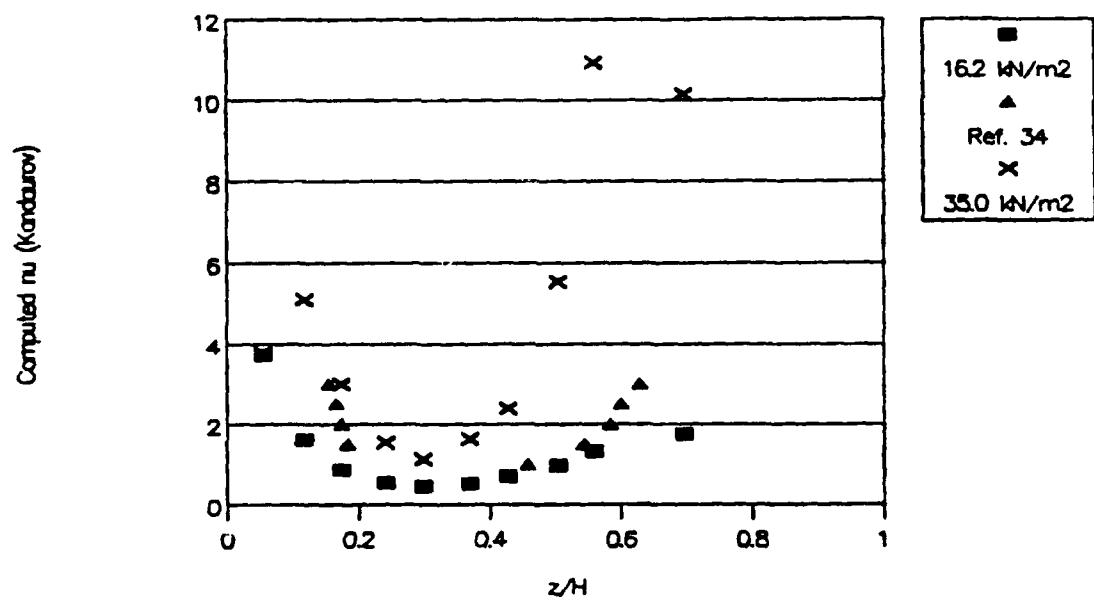
and  $\Phi(x) = \frac{1}{\sqrt{2\pi}} \cdot \int_0^x e^{-\frac{z^2}{2}} dz = \frac{2 \cdot \operatorname{erf}(\frac{x}{\sqrt{2}})}{\sqrt{2}}$

Figure 7 shows the observed normalized deflections from the model tests under the central axis of load and the corresponding computed deflections using equation 29, for different values of the coefficient of lateral stress,  $\nu$ . It is seen that the equivalent parameter  $\nu$  is not constant throughout the depth of the material under the load. Golden postulated that  $\nu$  is a power function of depth, or  $\nu = f(z^n)$ ; for reasons of dimensional compatibility Harr initially took  $n = 0$  which rendered it a constant, in a later development (Ref. 2 Chapter 9) he showed it varying with depth, much as is reported here.

Estimates were made of the nu-parameter,  $\nu$ , using measured deformations under the center of load,  $\Delta_z$ , for the initial loading of both MOBILE series, (i.e., 16.2 kN/m<sup>2</sup> and 35.0 kN/m<sup>2</sup>), and Kandaurov's equation 29. These estimates are shown in Figure 9.

Egorov's solution (in Figure 7) which was based on a rigid strip loading on a semi-infinite body, is shown for the bounding values of Poisson's ratios of  $\mu = 0$  and  $\mu = 0.5$ . It is seen that the observed deformations generally fall

Fig 9. Computed Nu



outside the prescribed band. However, in general the shape of the curves are similar in form except near the surface where they exhibit opposing curvatures.

Load kN/m <sup>2</sup>	Deformation (cm)	
	Loaded	Unloaded
0.0	0.0	0.0
3.8	5.6	5.4
7.2	6.0	5.9
11.0	7.3	7.3
14.8	8.6	8.5
22.0	11.1	11.0
38.1	15.8	15.8

Table 6. Contact Load versus Settlement

The reported "growth" of the bulb of vertical deformation with increasing load is smooth and well behaved, penetrating deeper into the model with each successively heavier load. An accompanying increase in the concentration of horizontal deformation under the edges of load is noted. In all cases no significant distortion was observed in the deformation field, and symmetry is remarkably well preserved throughout the loading and unloading processes. These suggest that the precautions taken to ensure uniform initial conditions were satisfactory.

#### Mobile Loading

In the two MOBILE loading series, the same general pattern of vertical deformation was observed as in the single STATIC tests, but they were "stretched" horizontally. This suggests that the resulting pattern of deformation could have been obtained by using a single loading plate taken to be as wide as the entire (mobile) loaded area. The distributions of horizontal deformations appear to be the same for the single loads, as for the "wider" loading plate except that the concentration of horizontal deformation

under the common edge (where the previous load would abut the current load location) is apparently erased: they appear to cancel each other out and the soil has no "memory" of previous loading

The peak magnitudes of the vertical and horizontal deformations under the single and "stretched" mobile loading arrangements are not materially different. The developing pattern of vertical deformation as the loading plate "expands" is clearly one-dimensional. This observation is in general agreement with the models of FREDLUND and RAHARDJO<sup>(32)</sup>, who used the concept and mechanics of consolidation to account for the deformation characteristics of granular materials.

The developed one-dimensional pattern of soil response to the moving load (Figures A25 thru A88) indicates that predictions of vertical deformations, may be obtained by scaling single load responses.

#### Surface deformation (settlement)

The recorded settlements of the soil surface under the loading plate were plotted against the magnitude of applied load in Figure 10. The plot demonstrates general linearity over the full test range. Note, however, that the graph does not pass through the origin (zero load). This is indicative of an initial pre-loading, or even a pre-compaction of the test material under its own weight prior to the first load application. The overall linearity of the graph suggests that surface settlement may be predicted from the methodology of a standard "Plate Bearing Test". Using the observed data in the conventional manner yields an equivalent uncorrected Modulus of Subgrade Reaction,  $k$ , of 12 psi/in.

POULOS<sup>(33)</sup> gave the expression for the settlement of the edge of a flexible strip loading on a layer of finite thickness as:

$$\rho_z = \frac{ph}{\pi E} \cdot I_{st}(\mu, h, b) \quad (26)$$

where  $p$  is the intensity of loading on the strip,  
 $h$  is the thickness of the loaded strip,  
 $E$  is Young's modulus of the soil,  
 $\mu$  is Poisson's Ratio of the soil,  
 $b$  is the width of the strip load,  
 $I_{st}$  is an influence factor.

He based this relationship upon classical arguments which require that Young's Modulus be constant, or at least independent of the magnitude of load. Equation 26 was used with the results of the model tests to calculate equivalent values of Young's Modulus. The results are shown in Figure 11 for two values of Poisson's ratio. It is seen that equivalent moduli do vary with loading and are not constant.

In Chapter 9 of Reference 2, Harr showed that Schmertmann's method of settlement analysis<sup>(34)</sup>, which was based on the theory of elasticity and the results of Eggestad<sup>(35)</sup>, could be improved by accounting for changes in the material state with depth. He demonstrated that superposing curves of normalized vertical strain,  $\epsilon_z E / \Delta p$ , for varying  $\nu$  values, he expanded upon Schmertmann's single triangle. Strains were obtained by applying a Hookean constitutive law to his probabilistic stress. It is quite remarkable that the values of  $\nu$  taken from the trace of Schmertmann's triangle provided such good agreement with those obtained from Kandaurov's equation. The overall shapes shown are in excellent agreement.

In Figure 12, the observed values of  $\nu$  from the model test are superimposed on a Schmertmann/Harr plot from Ref. 20. Once again it is seen that the overall agreement is quite graphic. It clearly demonstrates that the material state is not uniform once loaded (i.e.,  $\nu \neq \text{const.}$ ). In addition, directly under the load,  $\nu$  tends to be increasingly passive, while almost at an "at rest" ( $K_0$ ) condition is achieved at mid-depth. This is consistent with Harr's earlier methodology.

It is was observed through the entire progress of the testing program that the "rebound" of the material after removal of the loaded plate is negligible. Some comparisons are given in Table 6. The elastic (or recoverable) component of the material response is hardly differentiable.

Of historical note, plate bearing tests are used to provide predictions of soil settlement under load, but they do not provide any indication of response internal to the soil body. More sophisticated methods use the results from cone or screw penetrometers to provide profiles of material properties with depth. These more detailed data may be used in design methods (Terzaghi and Peck<sup>(36)</sup>, Peck, Hanson and Thornburn<sup>(37)</sup>, De Beer<sup>(38)</sup> and Schmertmann) to provide engineered estimates of settlement on and within the soil body. Whereas the Terzaghi and Peck and Peck, Hanson and Thornburn methods are based on bearing capacity methodology, and the De Beer method on consolidation mechanism, the Schmertmann method is based on the distribution of strain under the load. Harr recognized the applicability of probabilistic granular strain rather than the classical continuum strain to Schmertmann's method.

### Strains and Porosity

The strain information in Figures A1 thru A88 were obtained using "Green's measure of strain"<sup>(39)</sup> to compute the strains from the observed deformations. The "tear drop" pattern of horizontal strain is clear in both the STATIC and MOBILE cases, and is appears to be repeated with little modification with each successive load. The magnitudes of the horizontal strains are small, of the order of 2%, and of limited extent. The pattern of vertical strains emphasizes the one-dimensional aspect of vertical deformations reported earlier. Also strains are concentrated at the surface of the material under the load.

The analyses conducted of deformations and strains in the test model indicate that granular materials (as represented by the dry silt used) possess a "memory" that deformation "persists" after the removal of load. This is evidenced not only for applications of increasing load magnitude at the same location (STATIC), but in the case of the application of a constant load the model surface (MOBILE).

Fig 9. Computed Nu

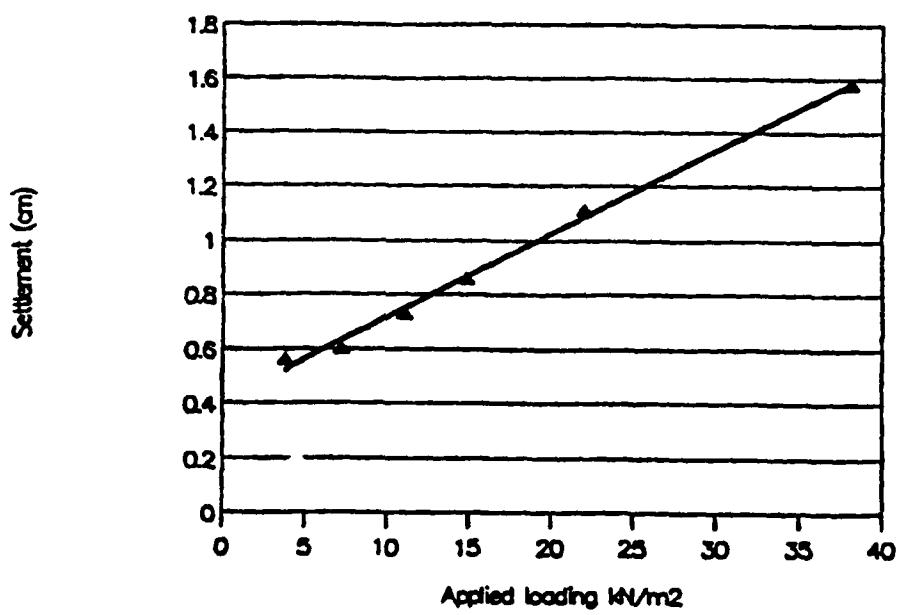
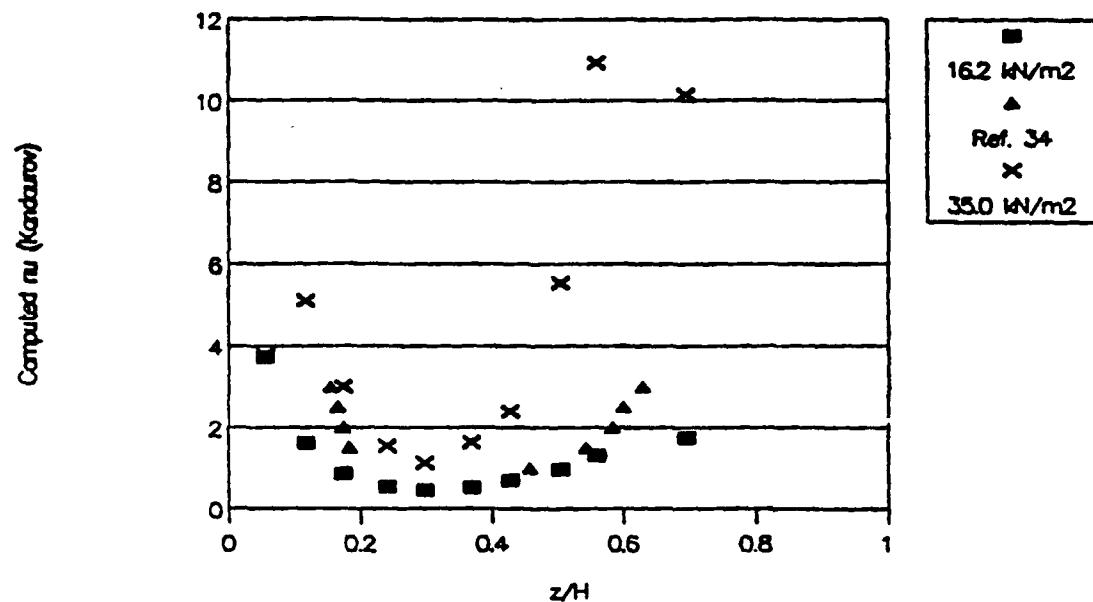
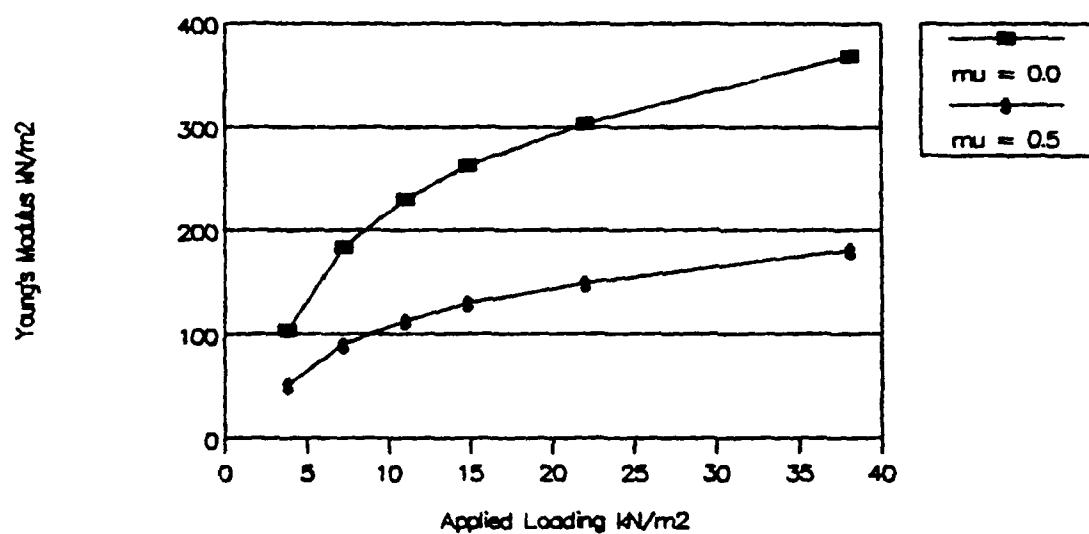


Figure 10 Measured settlement

Fig 11. Computed Moduli

(after Poulos)



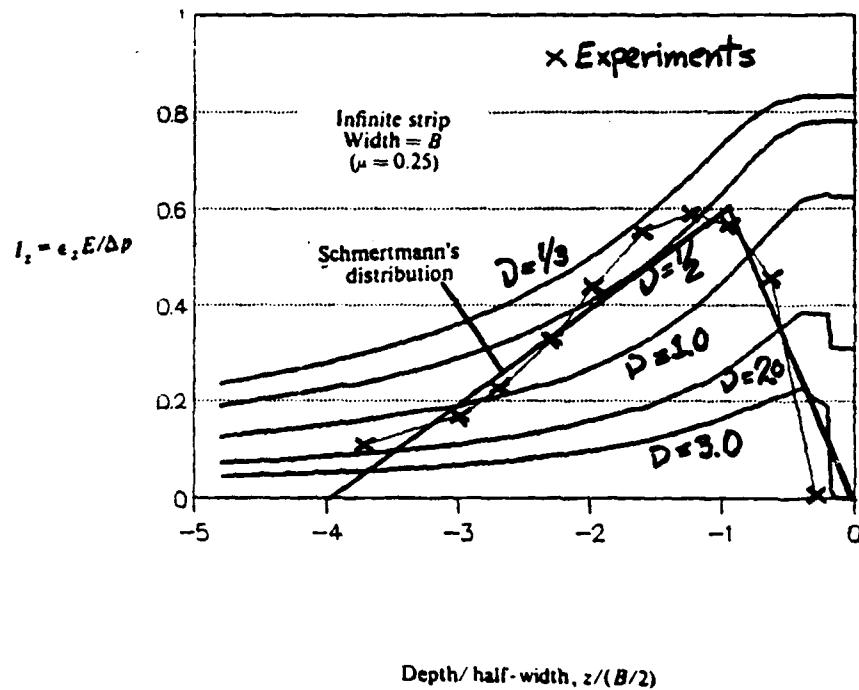


Figure 12 Schmertmann's distribution and superimposed NU-plots compared with experimental findings

## REFERENCES

- 1 BOURDEAU, P.L., "Analyse Probabiliste des Tassements d'un Massif de Sol Granulaire", Thesis No. 628(1986), Swiss Federal Institute of Technology, Lausanne, Switzerland (in French)
- 2 HARR, M.E., "Mechanics of Particulate Media - A Probabilistic Approach", McGraw-Hill, NY, 1977
- 3 LITWINISZYN, J., "The model of a random walk of particles adapted to researches on problems of mechanics of loose media", Bull. Acad. Polonaise des Sciences Techniques, Vol. 11, No. 10, 1963
- 4 ROWE, P.W., "The stress-dilatancy relation for static-equilibrium of particles in contact", Proc. Royal Society, London, Series A, Vol. 269, 1962
- 5 NEMAT-NASSER, S., "On Behavior of Granular Materials in Simple Shear", Soils and Foundations, Japanese Society of Soil Mechanics and Foundation Engineering, Vol. 20, No. 3, 1980
- 6 KITAMURA, R., "Analysis of Deformation Mechanism of Particulate Material at Particle Scale", Soils and Foundations, Japanese Society of Soil Mechanics and Foundation Engineering, Vol. 21, No. 2, 1981
- 7 ODA, M., "A Mechanical and Statistical Model of Granular Material", Soils and Foundations, Japanese Society of Soil Mechanics and Foundation Engineering, Vol. 14, No. 1, 1974
- 8 BOUSSINESQ, J., "Application des Potentiels à l'Étude de l'Équilibre et Mouvement des Solides Élastiques", Gauthier-Villard, Paris, 1885
- 9 FÖPPL, A., and FÖPPL, L., "Drang und Zwang", R. Oldenburg, Munich and Berlin, 1897
- 10 KÖGLER, D.P., and SCHEIDIG, A., "Baugrund und Bauwerk", Ernst und Sohn, Berlin, 1938
- 11 TURNBULL, W.J., MAXWELL, A.A., and AHLVIN, R.G., "Stresses and Deflections in Homogeneous Soil Masses", Proc. Fifth Int. Conf. Soil Mechanics and Foundations Engineering, Paris, 1961

- 12 GARRARD, C.M., and MULHOLLAND, P., "Stress, Strain and Displacement in Cross-Anisotropic and Two Layer Isotropic Elastic Systems", Proc. Third Conf. Australian Road Research Board, 1966
- 13 Van CAUWELAERT, F. "L'Élasticité Anisotrope Appliquée à la Méchanique des Milieux Granulaires et des Roches", Thesis, Swiss Federal Institute of Technology, Lausanne, Switzerland, 1983
- 14 DUNCAN, J.M. and CHANG, C.Y., "Nonlinear Analysis of Stress and Strain in Soils", Jnl. Soil Mechanics and Foundations Division, ASCE, Vol. 96, No. SM5, 1970
- 15 JANBU, N., "Soil Compressibility as Determined by Oedometer and Triaxial Tests", Proc. European Conf. Soil Mechanics and Foundation Engineering, Wiesbaden, Vol. 1, 1963
- 16 LADE, P.V., "The Stress-Strain and Strength Characteristics of Cohesionless Soils", Thesis, University of California, Berkeley, 1972
- 17 POKROVSKI, G.I., "Investigation on the Physics of Soils", Vodgeo, ONTI, 1937 (in Russian)
- 18 KANDAUROV, I.I., "Theory of Discrete Distribution of Stress and Deformation", Izd. VATT. 1959 (in Russian)
- 19 MULLER, R.A., "Statistical Theory of Stress Distribution in a Granular Soil Foundation Bed", Bases, Foundations and Soil Mechanics, Vol. 4, No. 4, 1962 (in Russian)
- 20 SERGEEV, I.T., "The Application of Probability-Processes Equations to the Theory of Stress Distribution in Non-Cohesive Soil Foundation Beds", Soil Mechanics and Foundation Engineering, No. 2, 1969
- 21 SMOLTCZYK, H.U., "Stress Computation in Soil Media", Jnl. Soil Mechanics and Foundations Division, ASCE, Vol 93, No. SM2, 1867
- 22 ENDLEY, S.N. and PEYROT, A.H., "Load Distribution in Granular Media", Jnl. Soil Mechanics and Foundations Division, ASCE, Vol. 103, No. EM1, 1977
- 23 GOLDEN, J.M., "Stochastic Models of Granular Media", Jnl. Engineering Mechanics Division, ASCE, Vol. 110, No. EM1, 1984
- 24 REIMBERT, M.L. and REIMBERT, A.M., "Retaining Walls", Trans. Technology Publishing, Bay Village, Ohio, 1974

- 25 MARSAL, R.J., "A Statistical Model for Granular Materials", Proc. 7<sup>th</sup> ICSFME, Specialty Session 13, Mexico, 1969
- 26 SWEET, A.L., and BOGDANOFF, J.L., "Stochastic Model for Predicting Subsidence", Jnl. Engineering Mechanics Division, ASCE, Vol. 91, No. EM2, 1965
- 27 MARSAL, A.J., "Particle Breakage in Coarse Granular Soils", Proc. 7<sup>th</sup> ICSFME, Mexico, 1969
- 28 CRC, Handbook of Physics and Chemistry
- 29 Rourke, R., "Stress Handbook"
- 30 MIKHAIL, E.M., and ACKERMANN, F.E., "Observations and Least-Squares", IEP, 1976
- 31 EGOROV, K.E., "Distribution of Stresses in Base under Rigid Strip Footing", Nauchn. Issled, Stanstiya Fundamentstroya, No. 9, 1940
- 32 FREDLUND, D.G., and RAHARDJO, H., "Unsaturated Soil Consolidation Theory and Laboratory Experimental Data", Consolidation of Soils: Testing and Evaluation, ASTM STP 892, 1986
- 33 Poulos, H.G., and DAVIS, E.H., "Elastic Solutions for Soil and Rock Mechanics", John Wiley and Sons, NY, 1974
- 34 SCHMERTMANN, J.H., "Static Cone to Compute Settlement Over Sand", Jnl., Soil Mechanics and Foundations Division, ASCE, Vol. 96, SM3, 1970
- 35 EGGESTAD, A., "Deformation Measurements Below a Model Footing on the Surface of Dry Sand", Proc. European Conf. Soil Mechanics and Foundation engineering, Vol. 1, Wiesbaden, 1963
- 36 TERZAGHI, K., and PECK, R.B., "Soil Mechanics in Engineering Practice", 2<sup>nd</sup> Ed. John Wiley and Sons, NY, 1967
- 37 PECK, R.B., HANSON, W.E., and THORNBURN, T.H., "Foundation Engineering", 2<sup>nd</sup> Ed., John Wiley and Sons, NY, 1974
- 38 De BEER, E.E., "Bearing Capacity and Settlement of Shallow Foundations on Sand", Proc. Symposium on Bearing Capacity and Settlement of Foundations, Duke University, 1965

39 FENNER, R.T., "Engineering Elasticity", John Wiley and Sons, NY

40 SCHULITZE, E., "Frequency Distributions and Correlations of Soil Properties", Statistics and Probability in Civil Engineering, Hong Kong University Press, Distributed by Oxford University Press, 1972

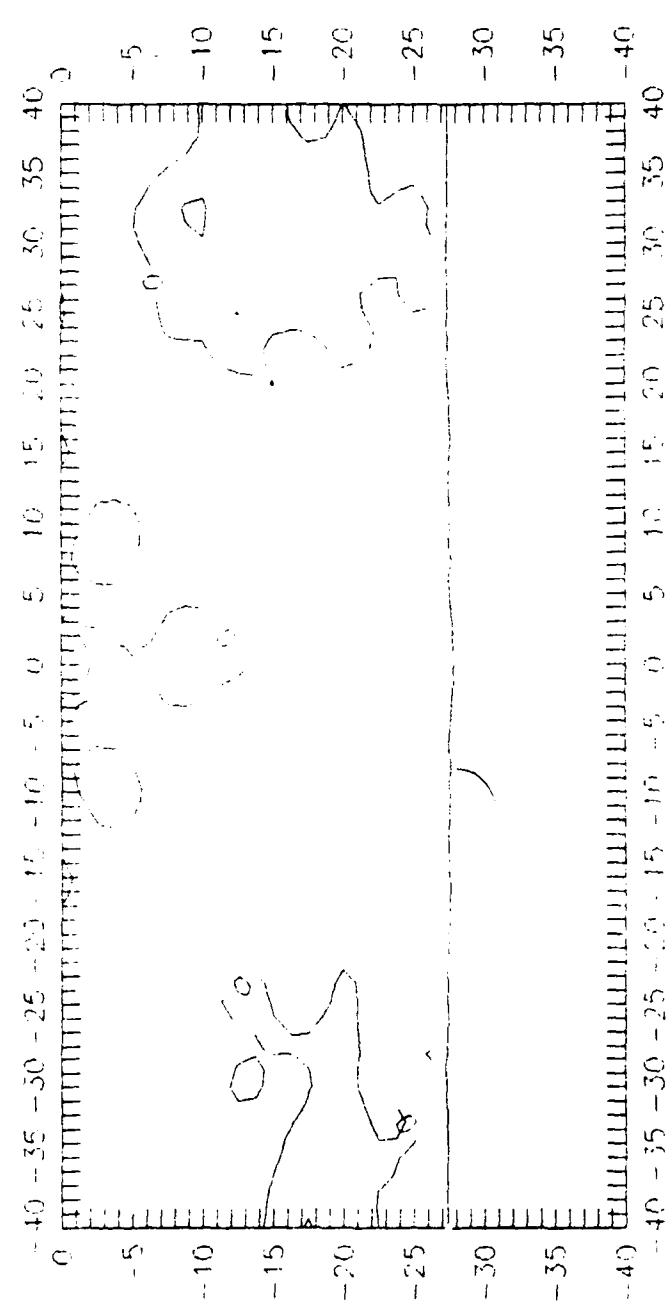
41 GRIVAS, D.A., and HARR, M.E., "Particle Contacts in discrete Materials", Abstracts of the Indiana Academy of Sciences, Engineering Division, 1975

42 ROHTAGI, V.K., "An Introduction to Probability Theory and Mathematical Statistics", John Wiley and Sons, NY, 1976

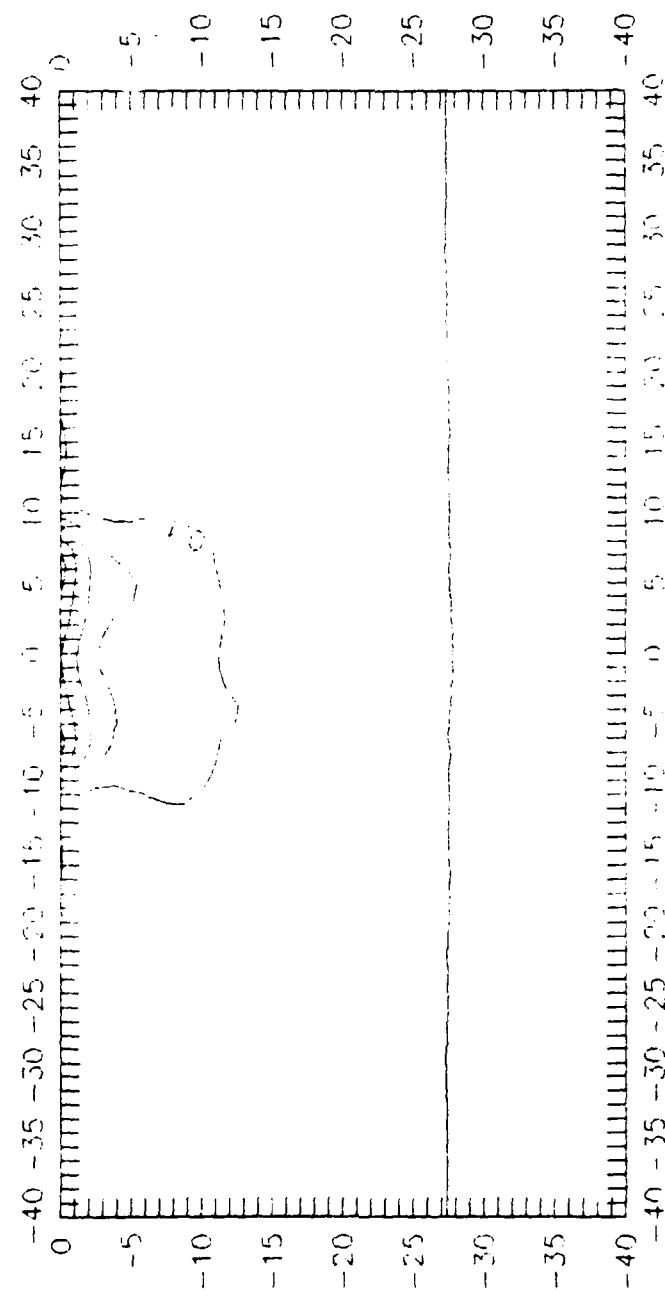
## APPENDIX A

A1 thru A24	STATIC Series
A25 thru A54	MOBILE Series
	Load 16.2 kN/m <sup>2</sup>
A56 thru A85	MOBILE Series
	Load 35.0 kN/m <sup>2</sup>

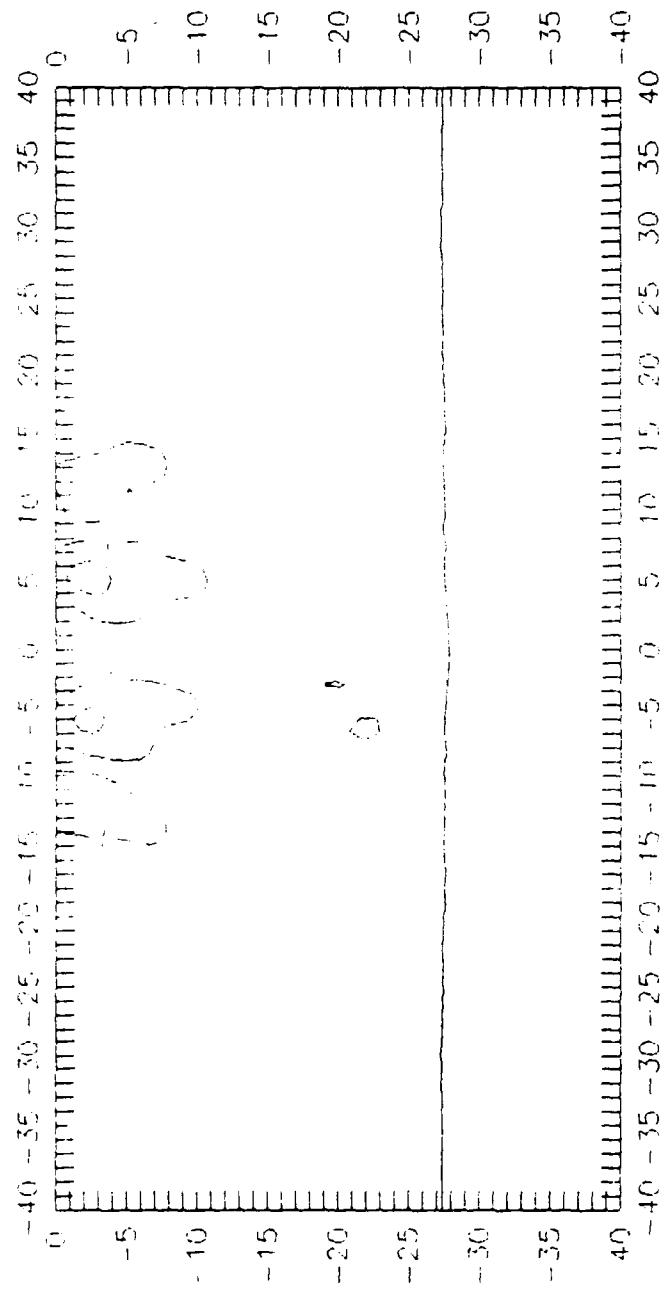
STATIC =  $3\epsilon$  (11.7m) = Horizontal Deflection



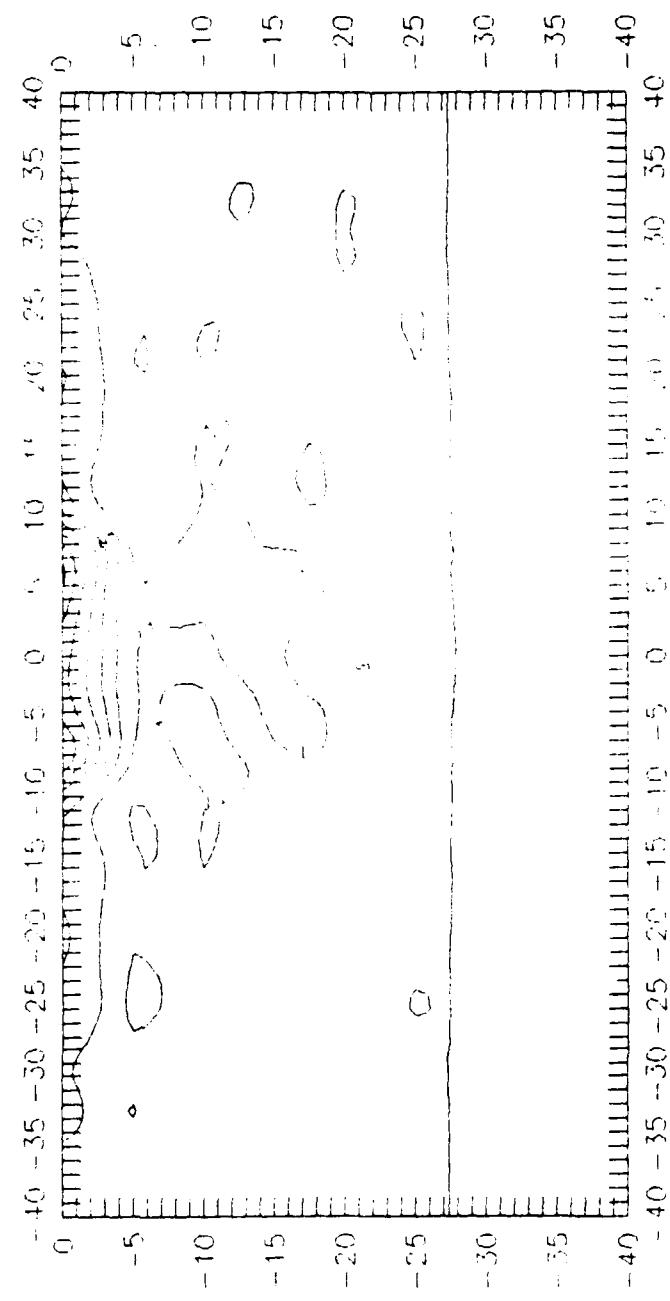
STATIC 3.8  $\mu\text{m}^2$  = Vertical Conformation

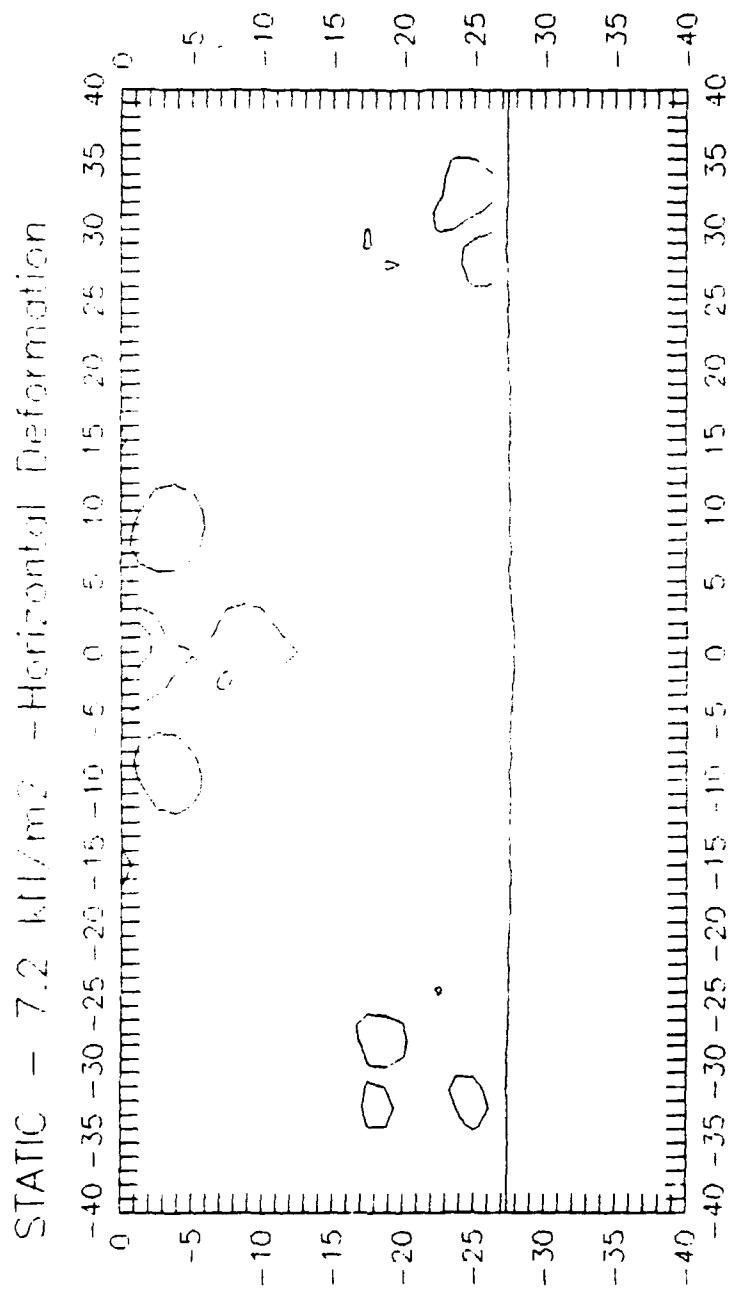


STATIC 3.8 (11, 012) - 1100 ft. right side (37)

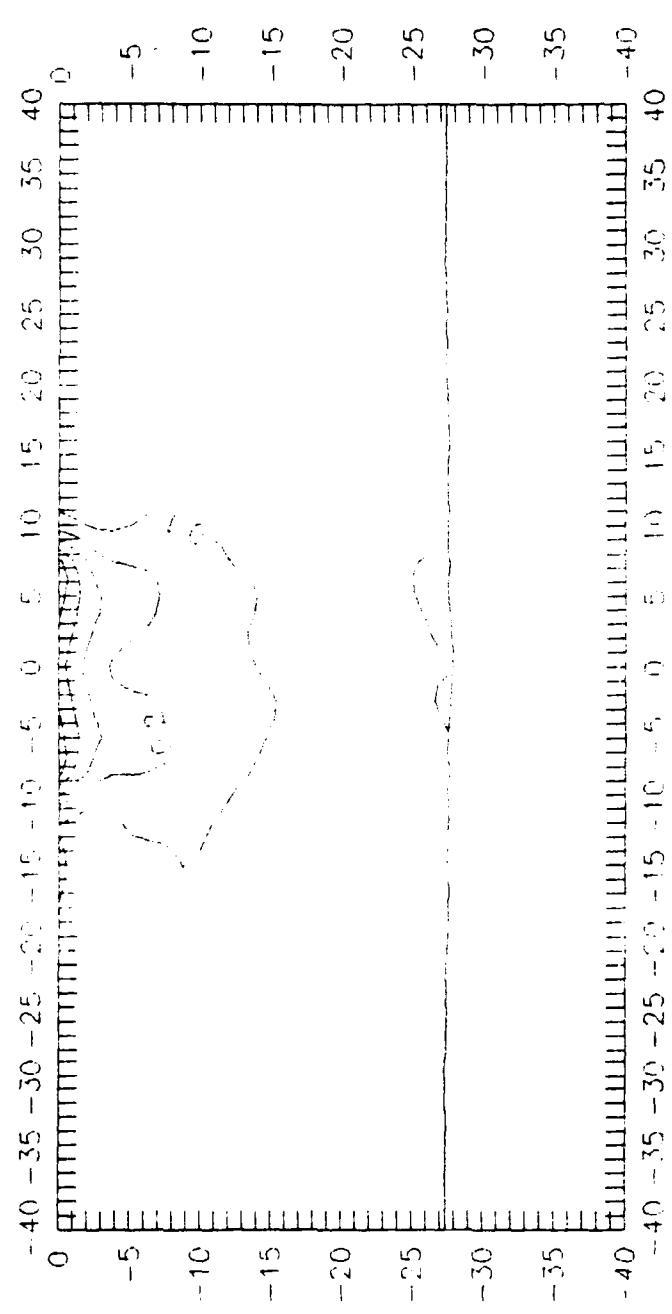


STATIC 3.8 Wind on Optimal Stair (31)

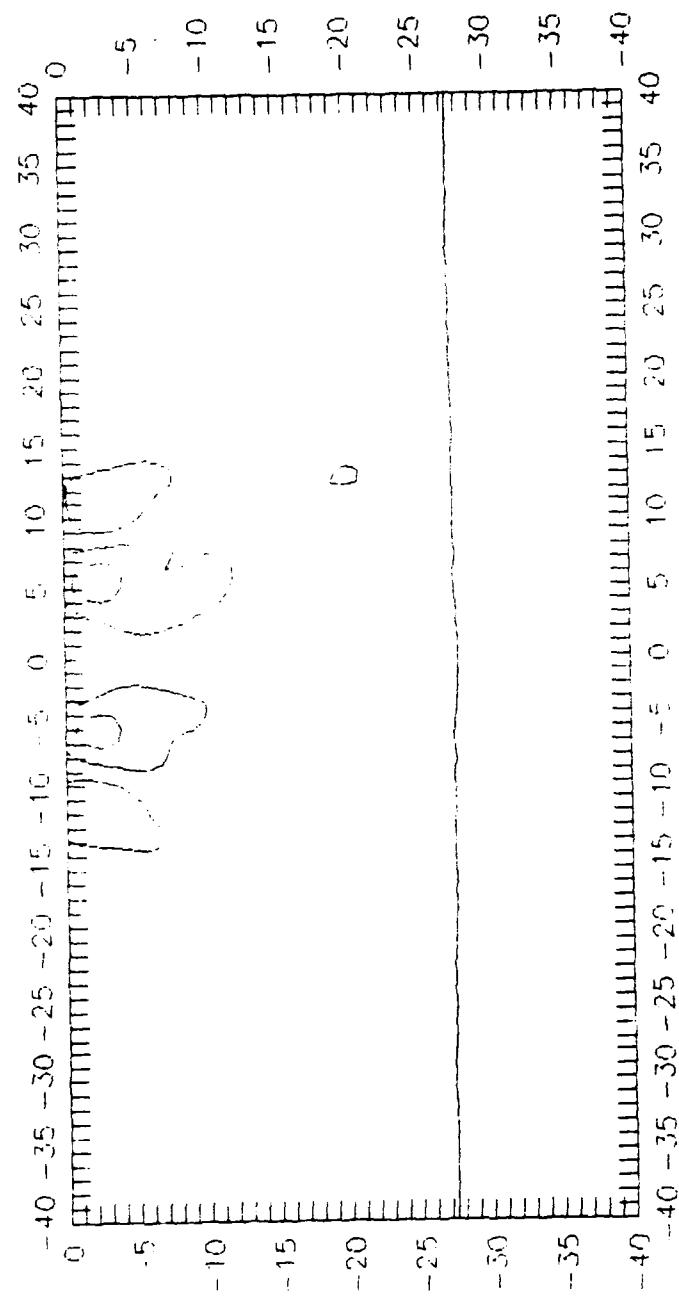


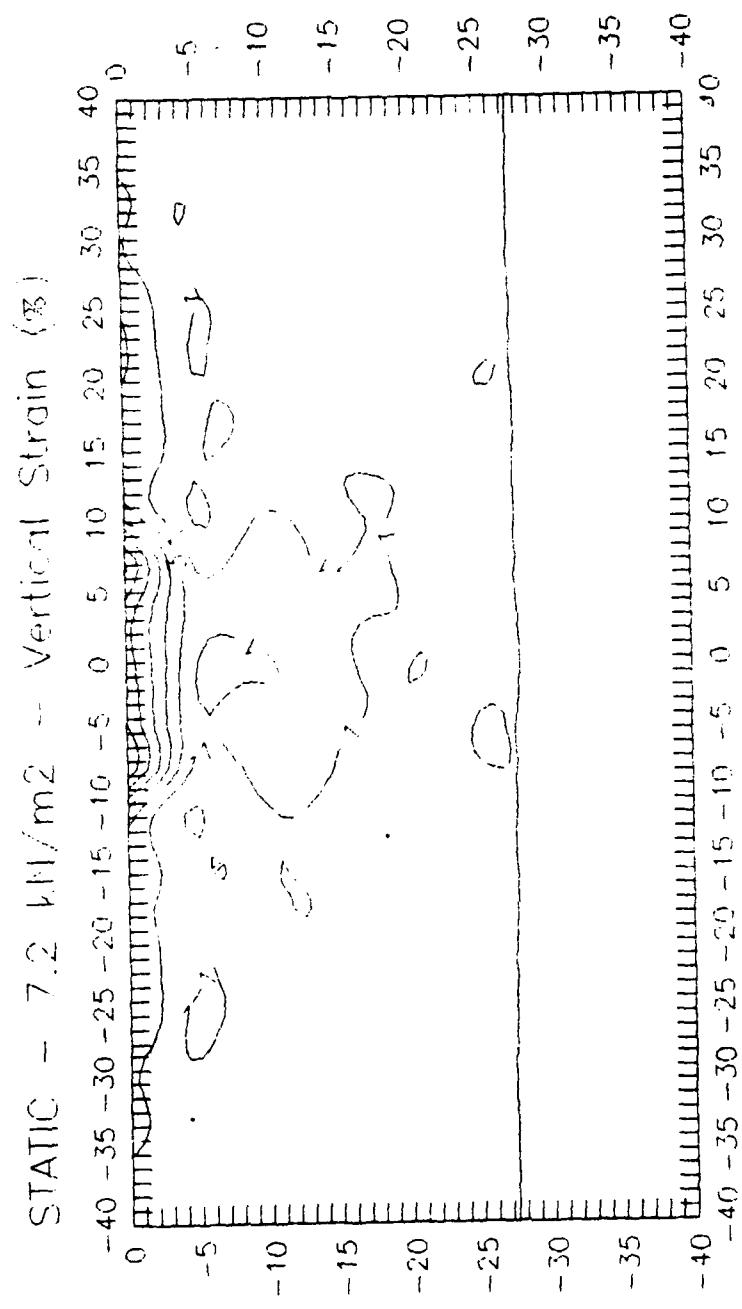


STATIC = 7.2 111/m<sup>2</sup> = vertical deflection

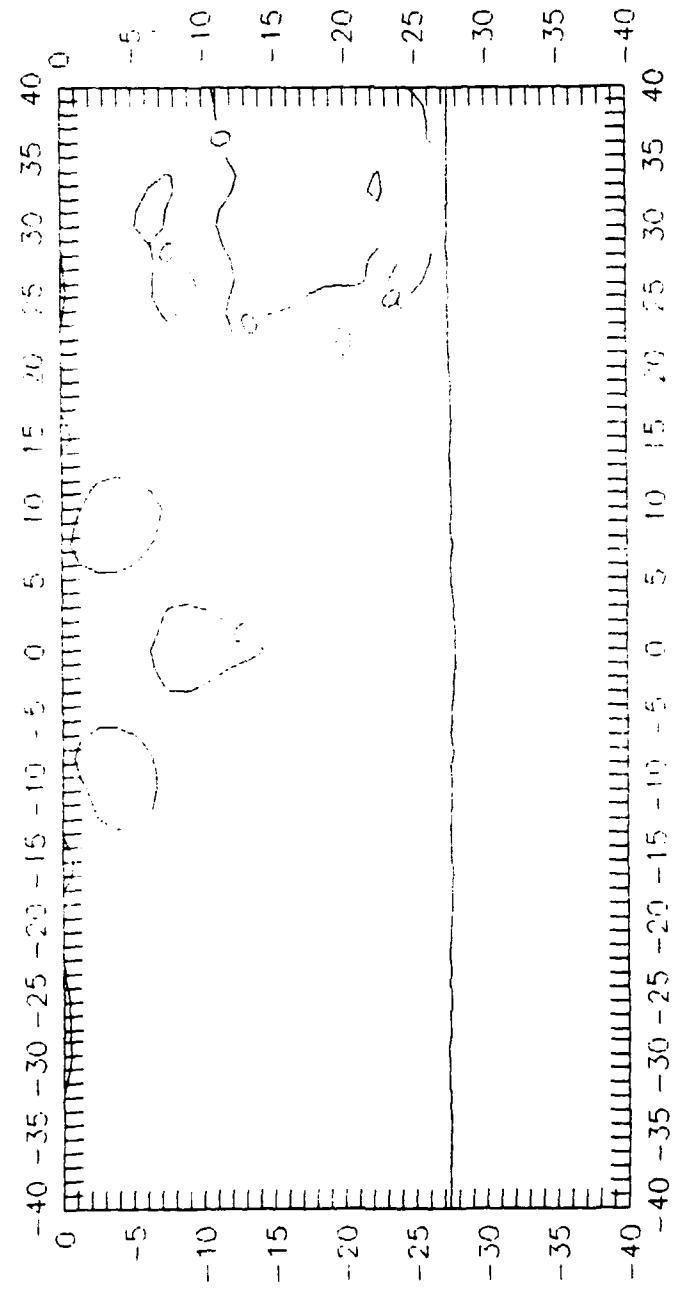


STATIC = 7.2 kN/m<sup>2</sup> = Horizontal Strain (%)

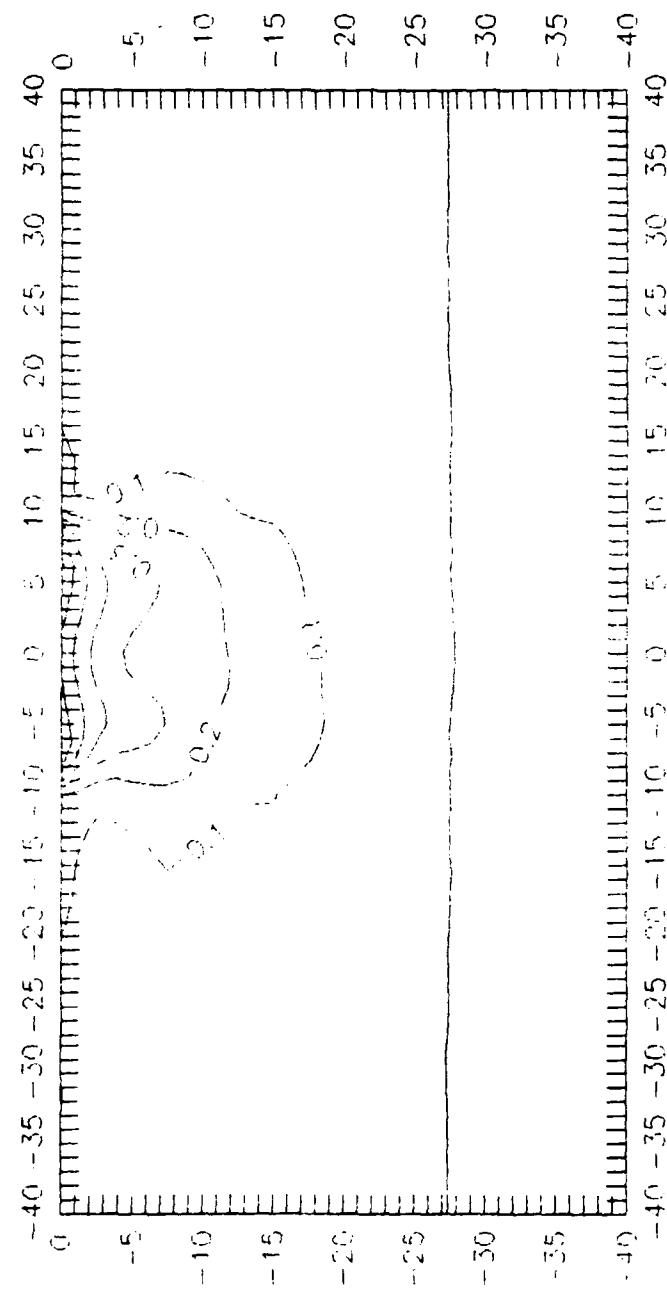


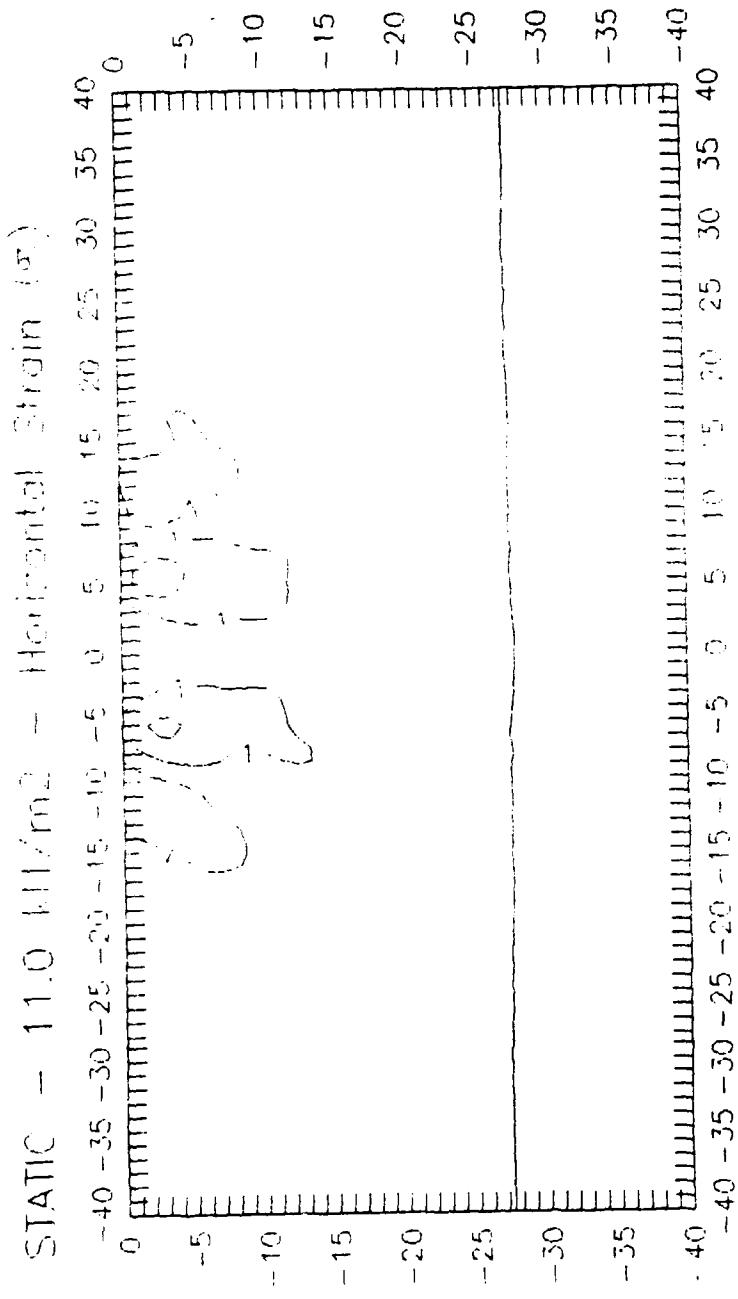


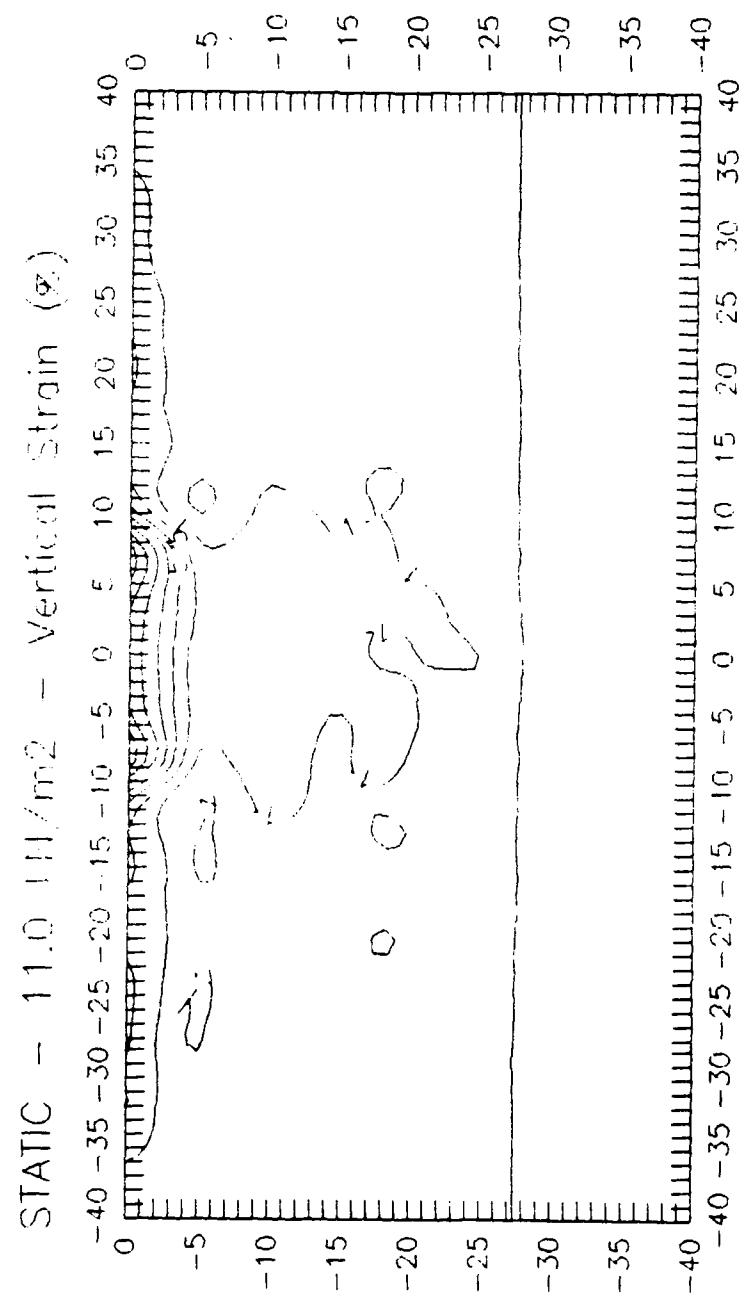
STATIC = 11.0  $\text{ft}^2/\text{m}^2$  = Horizontal Deformation



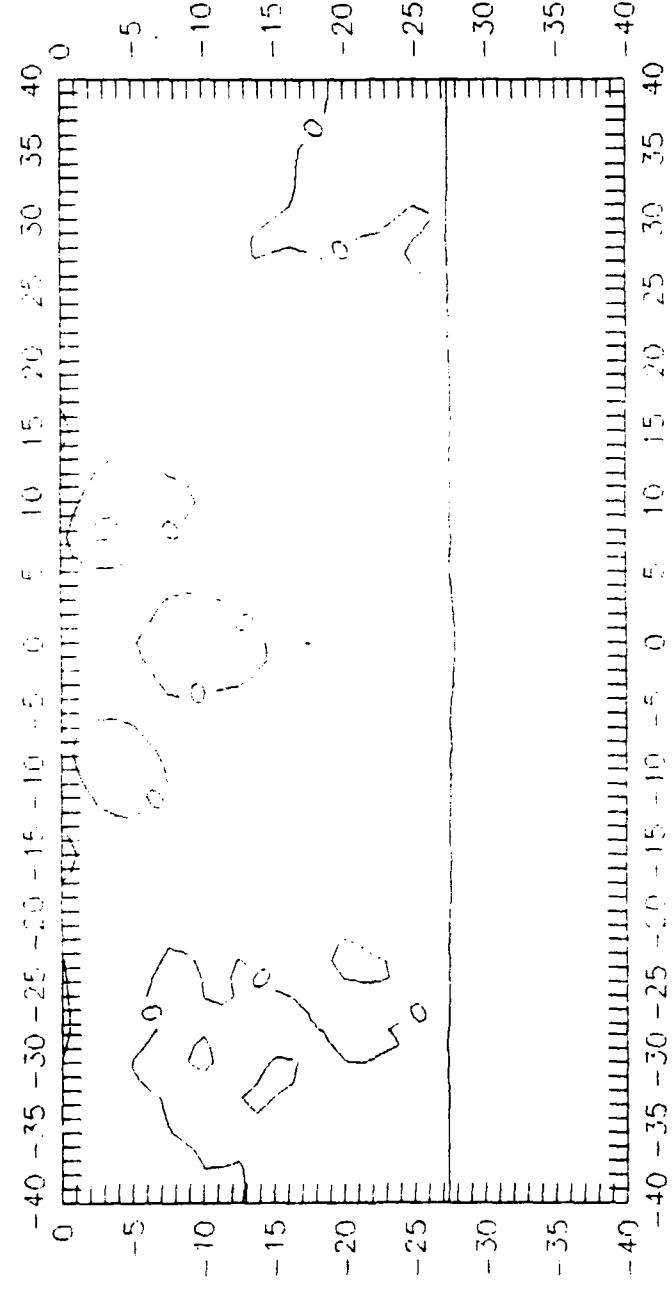
STATIC = 11.C111.002 = vertical Deformation



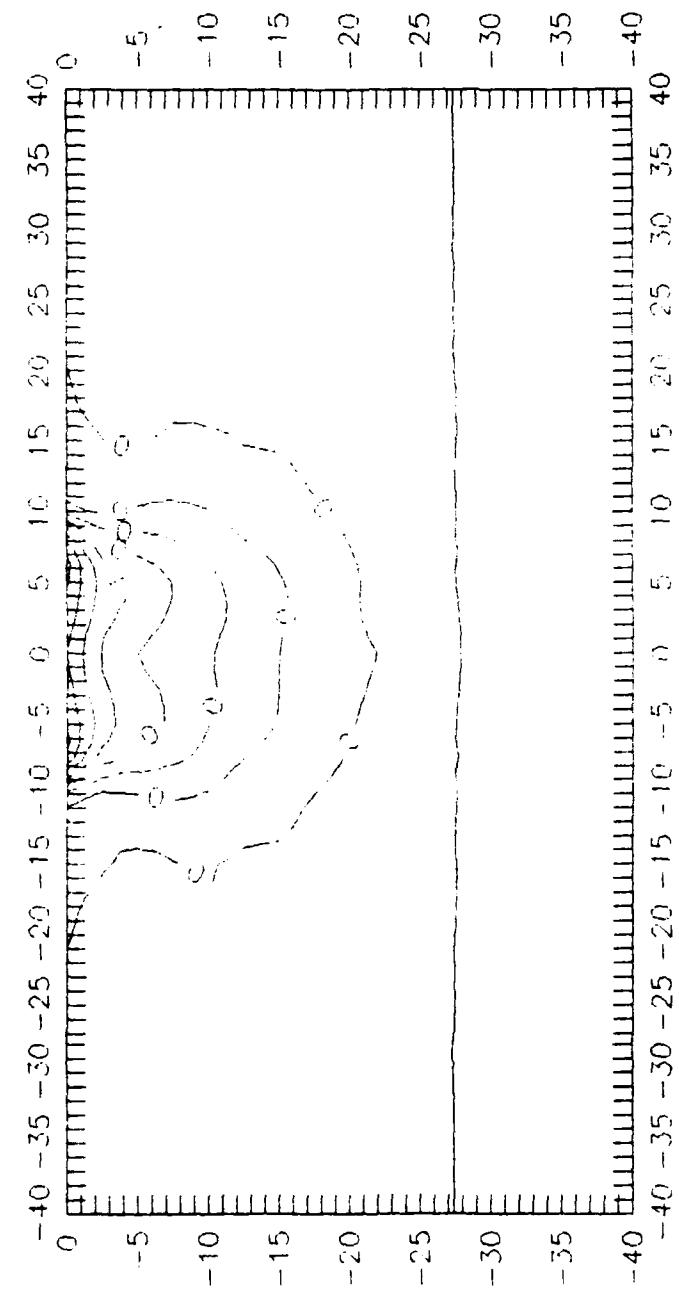




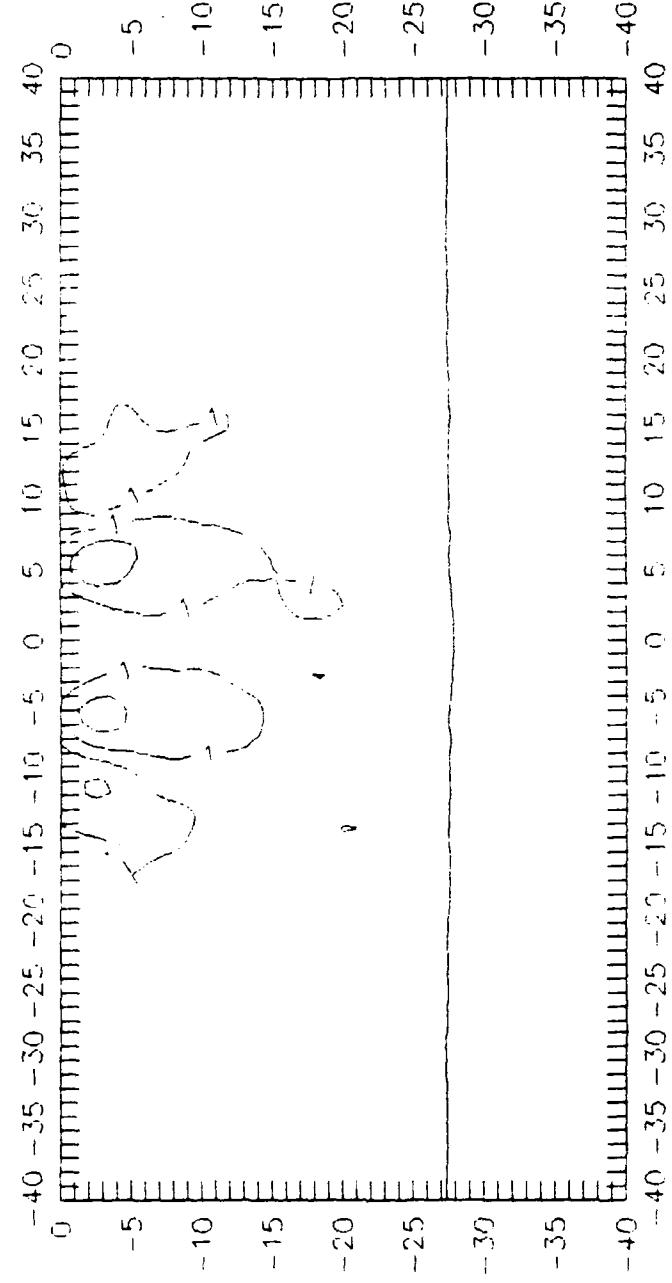
STATIC = 14.8 kN/m<sup>2</sup> = Horizontal Deformation



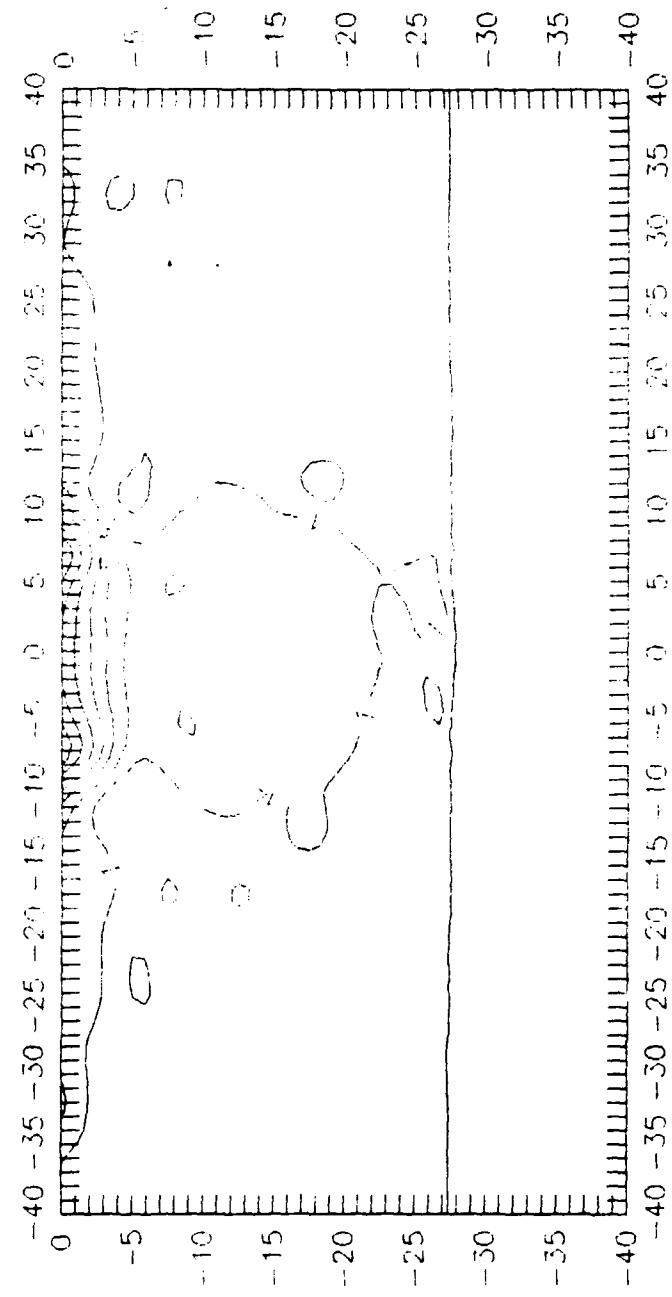
STATIC = 14.8 kN/m<sup>2</sup> = Vertical deformation



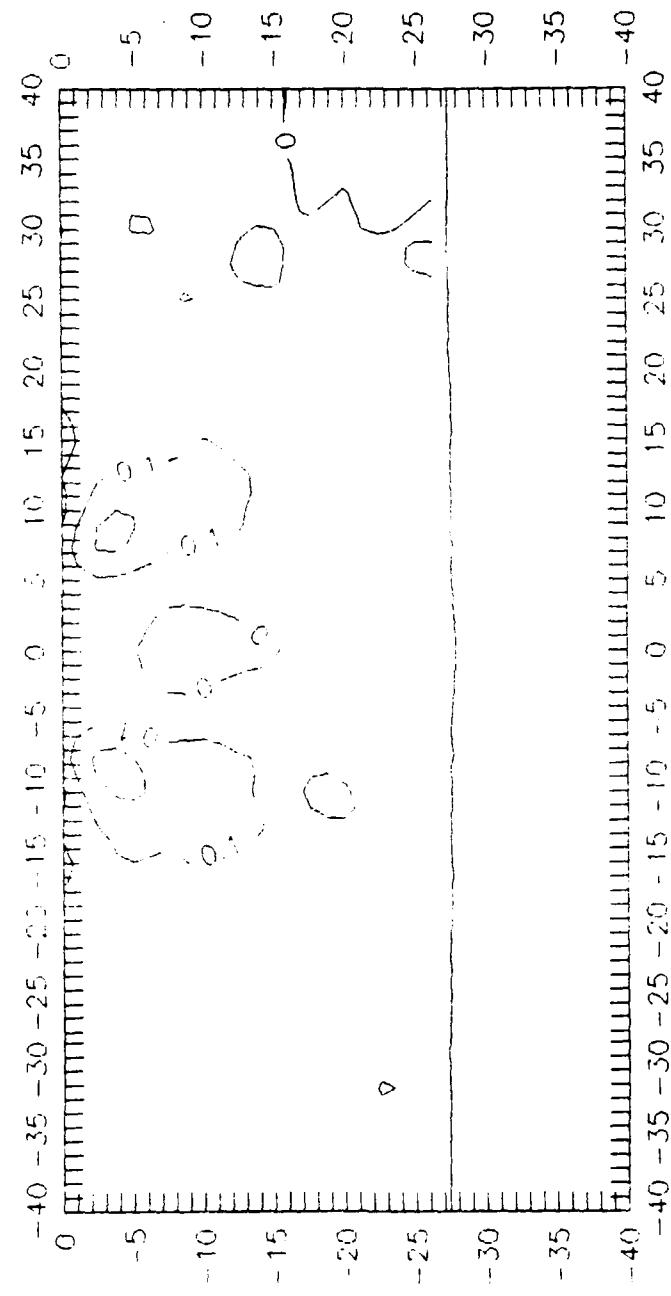
STATIC = 14.8 kN/m<sup>2</sup> = Horizontal Strain ( $\epsilon_x$ )



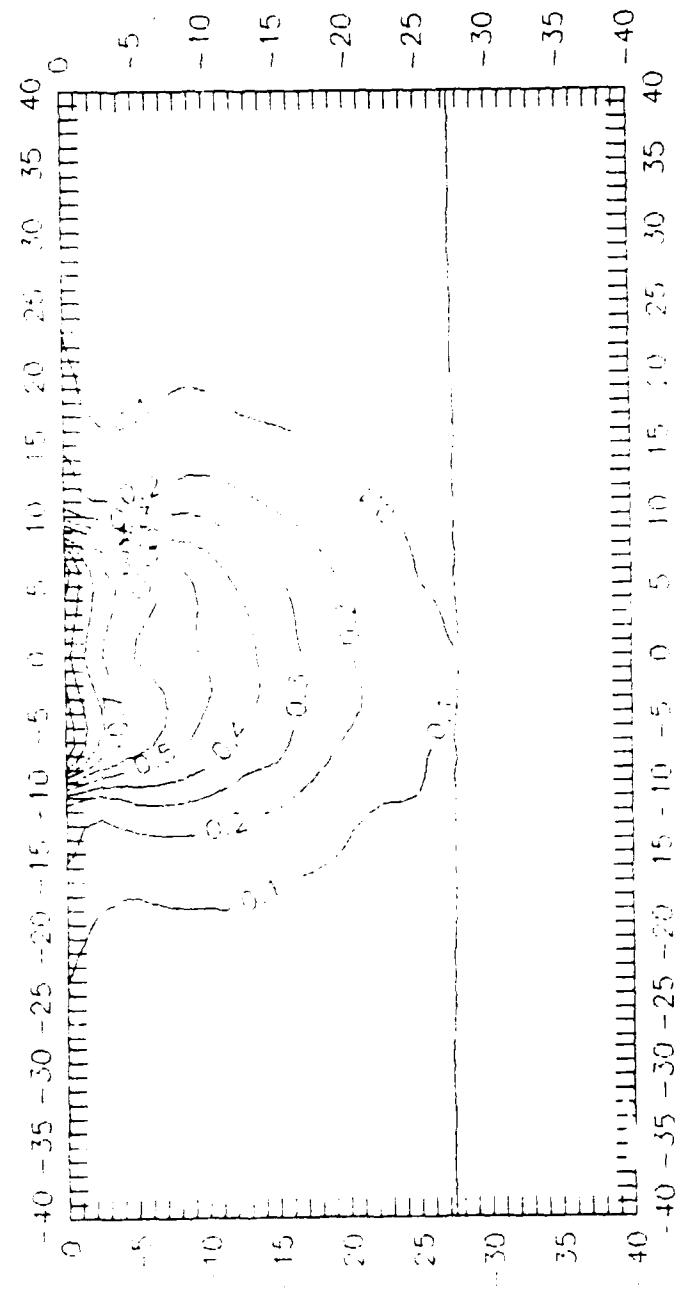
STATIC - 14.8 kNm<sub>2</sub> - Vertical Margin ( $\sigma_v$ )



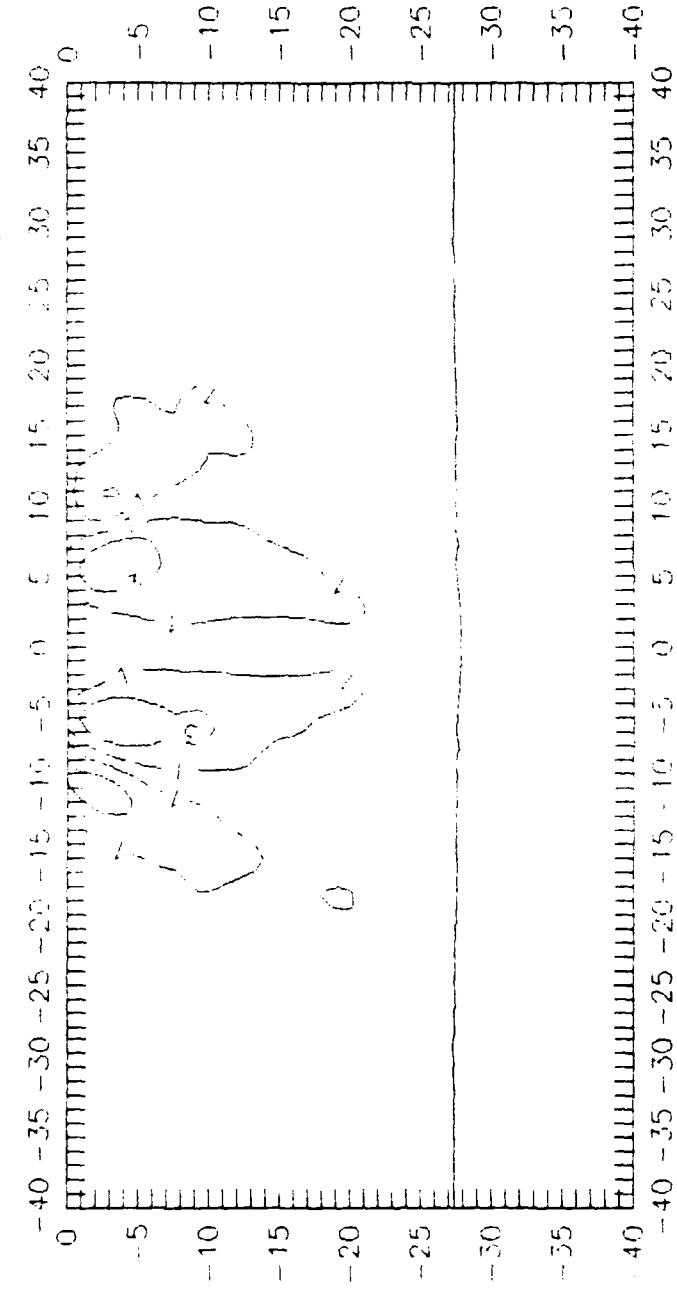
STATIC = 22.0 kN/m<sup>2</sup> - Horizontal Deformation

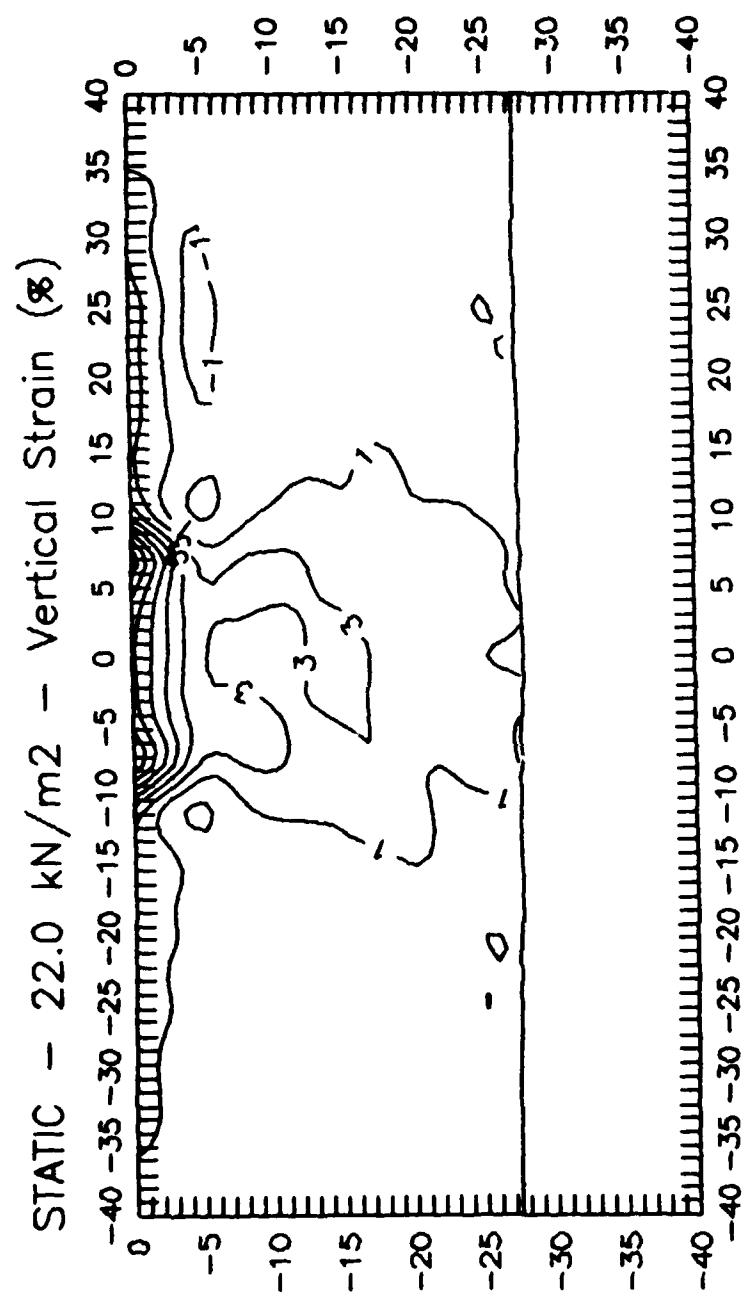


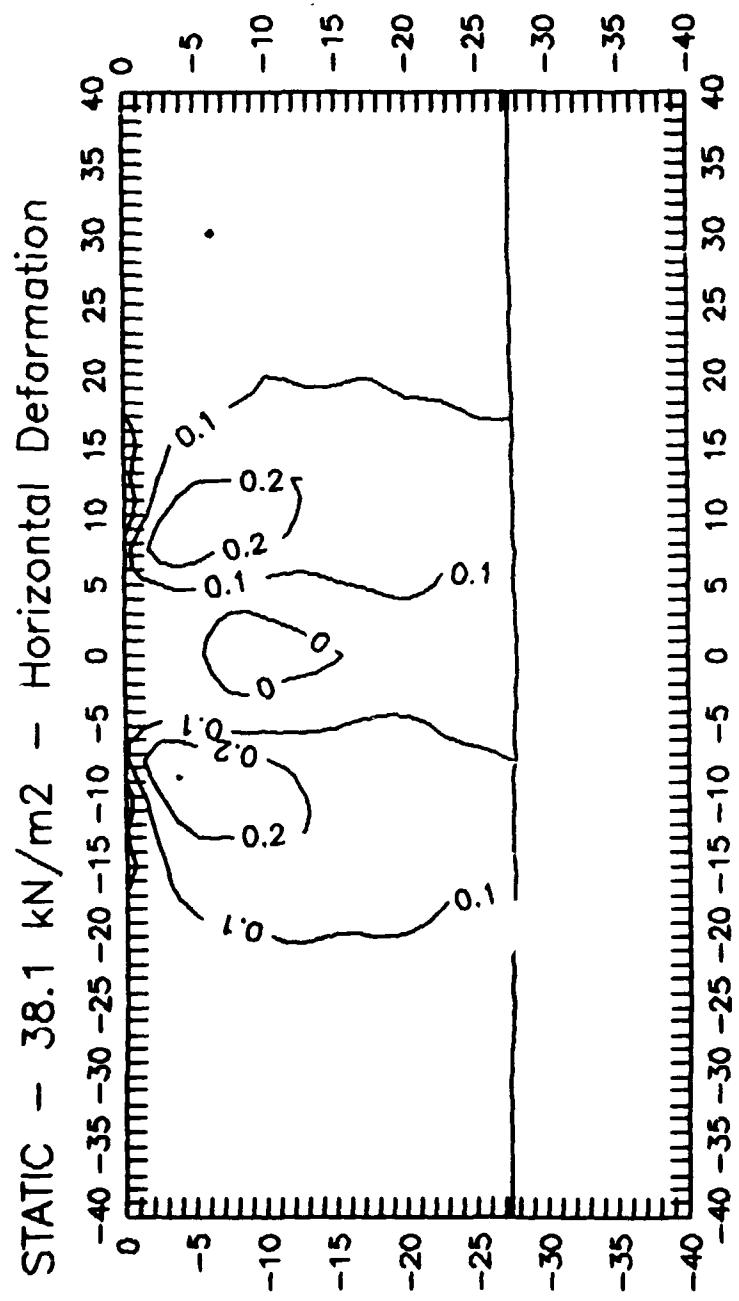
STATIC = 22.0  $\mu$ strain Vertical deformation

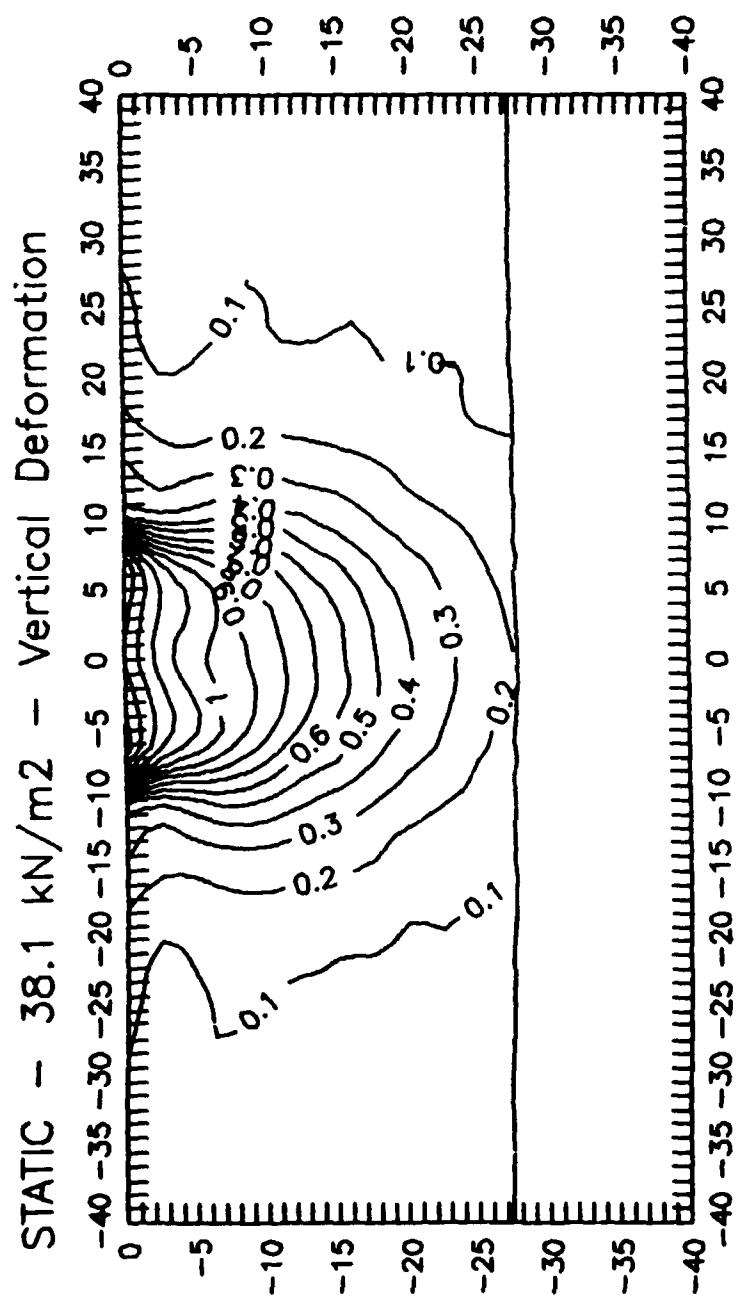


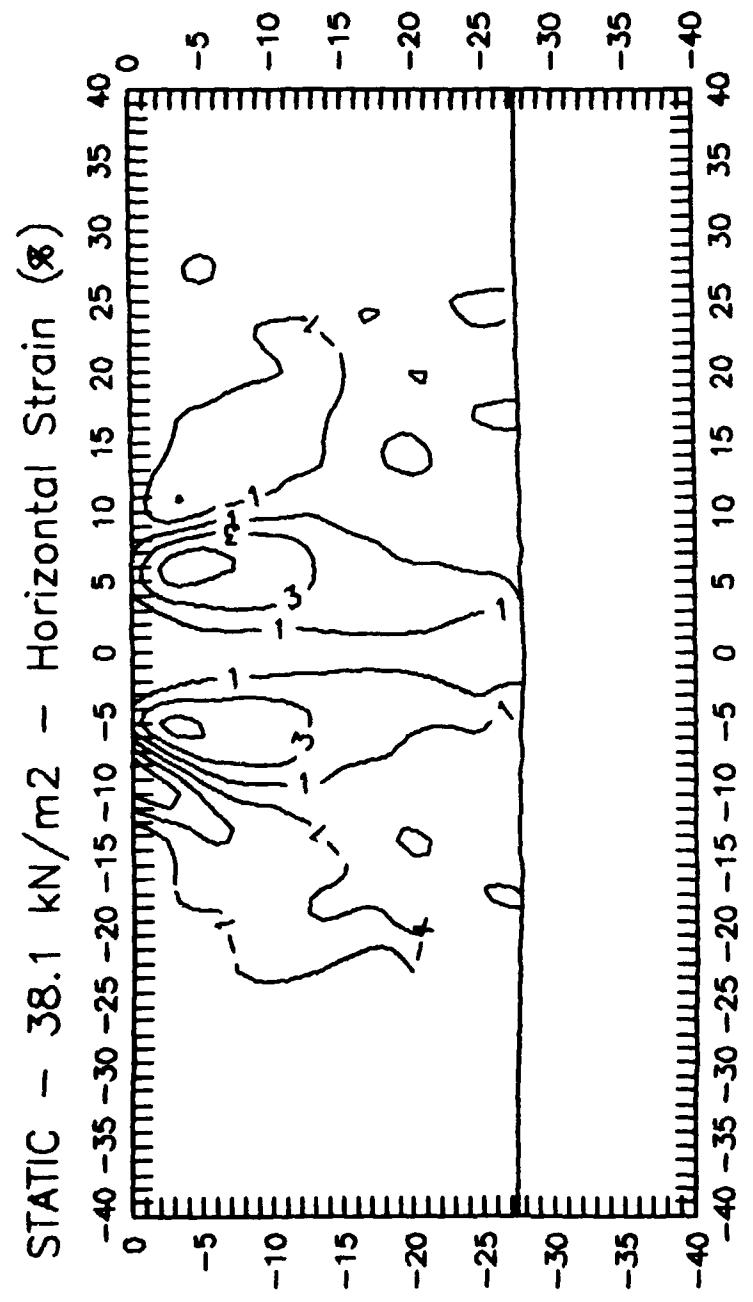
STATIC = 22.0  $\text{lb/in}^2$  = Horizontal Stress (psi)

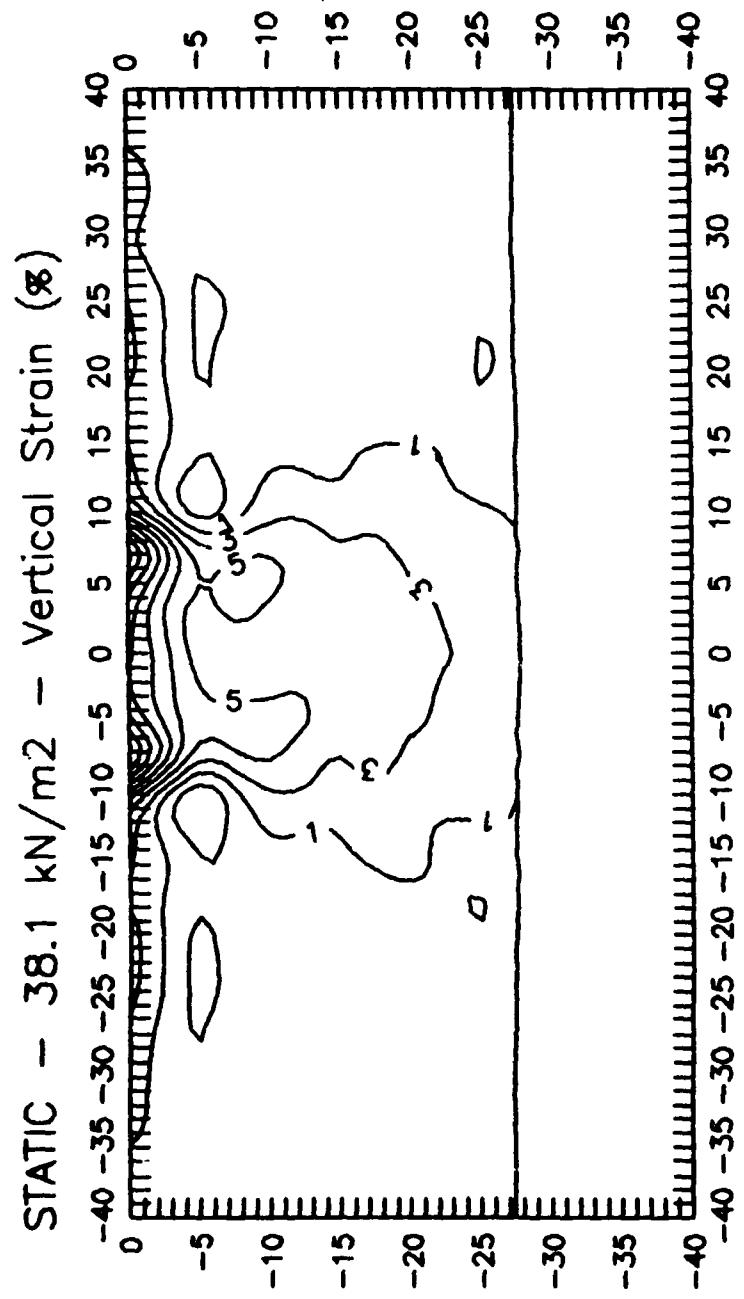








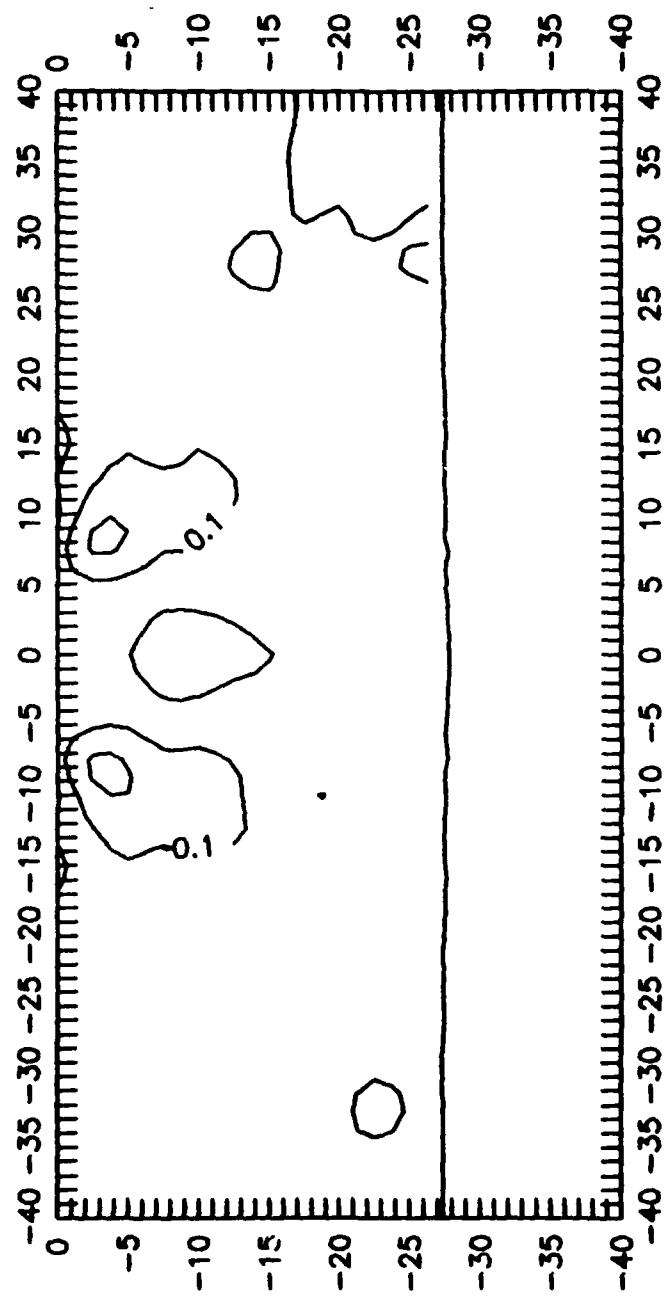


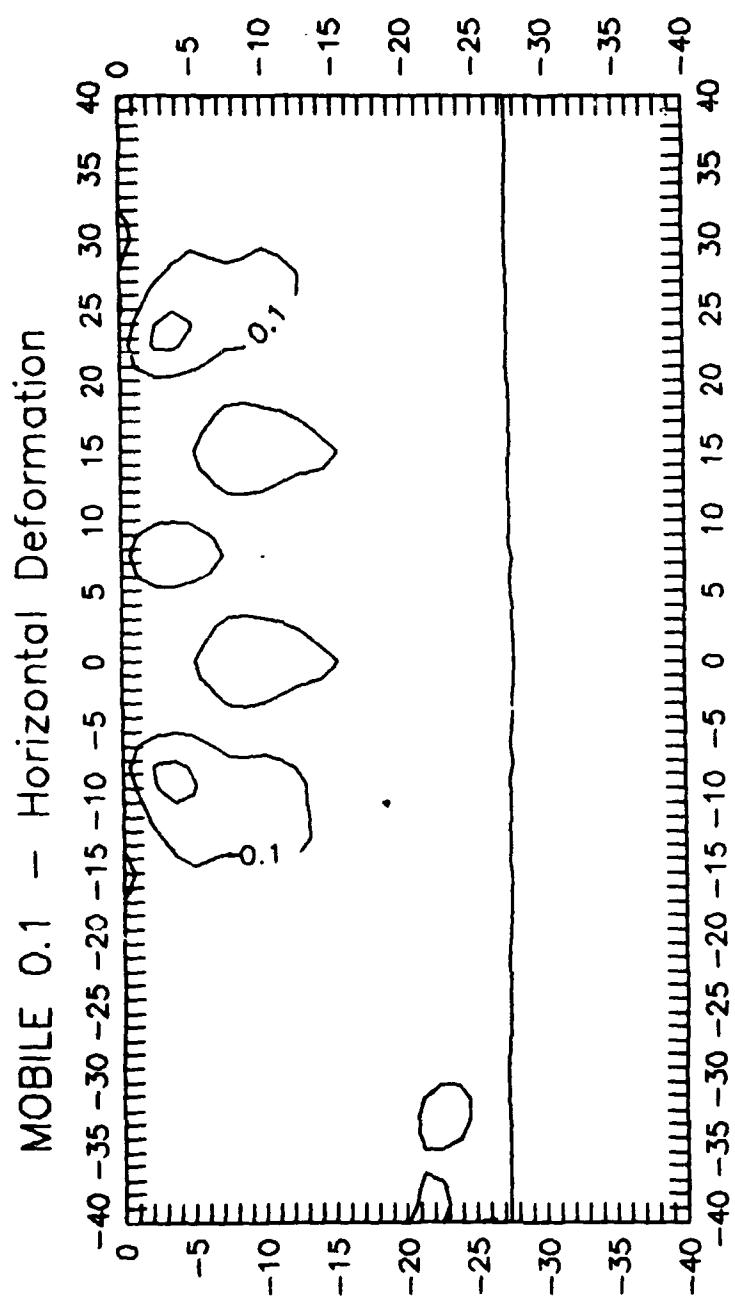


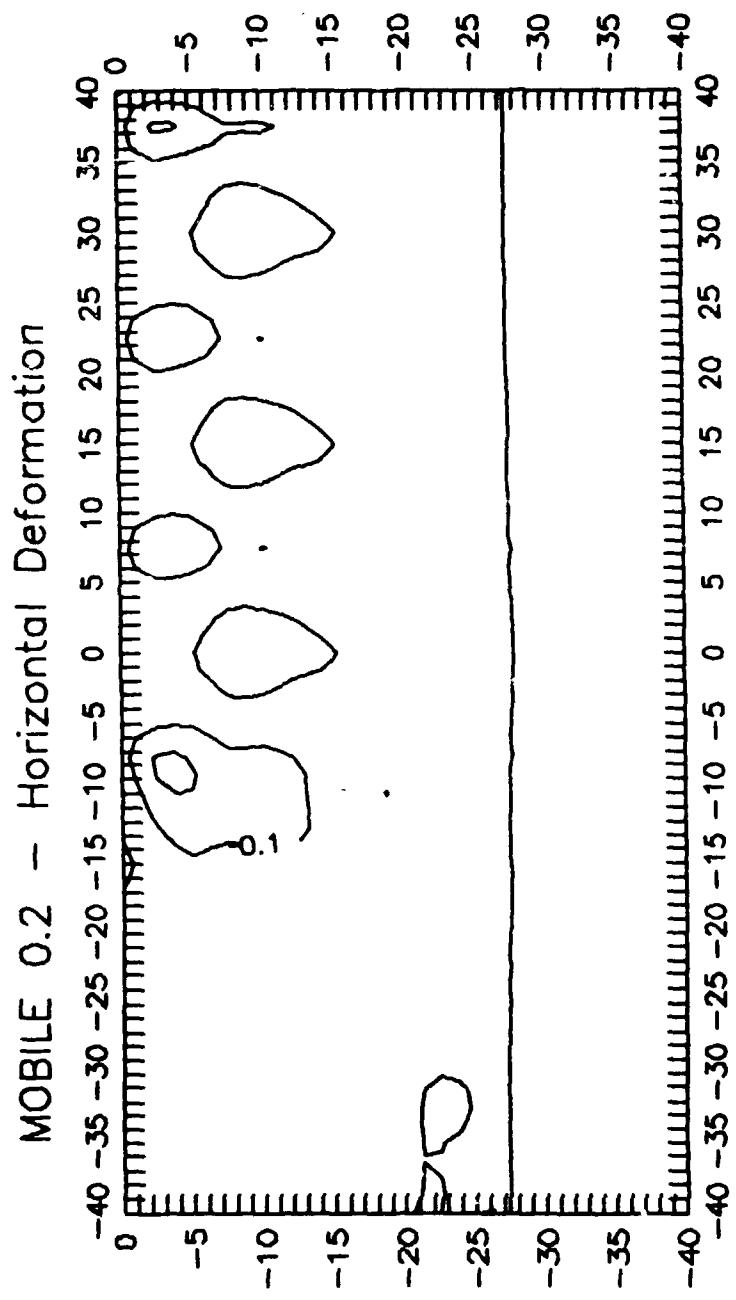
MOBILE SERIES

Load 16.2 kN/m<sup>2</sup>

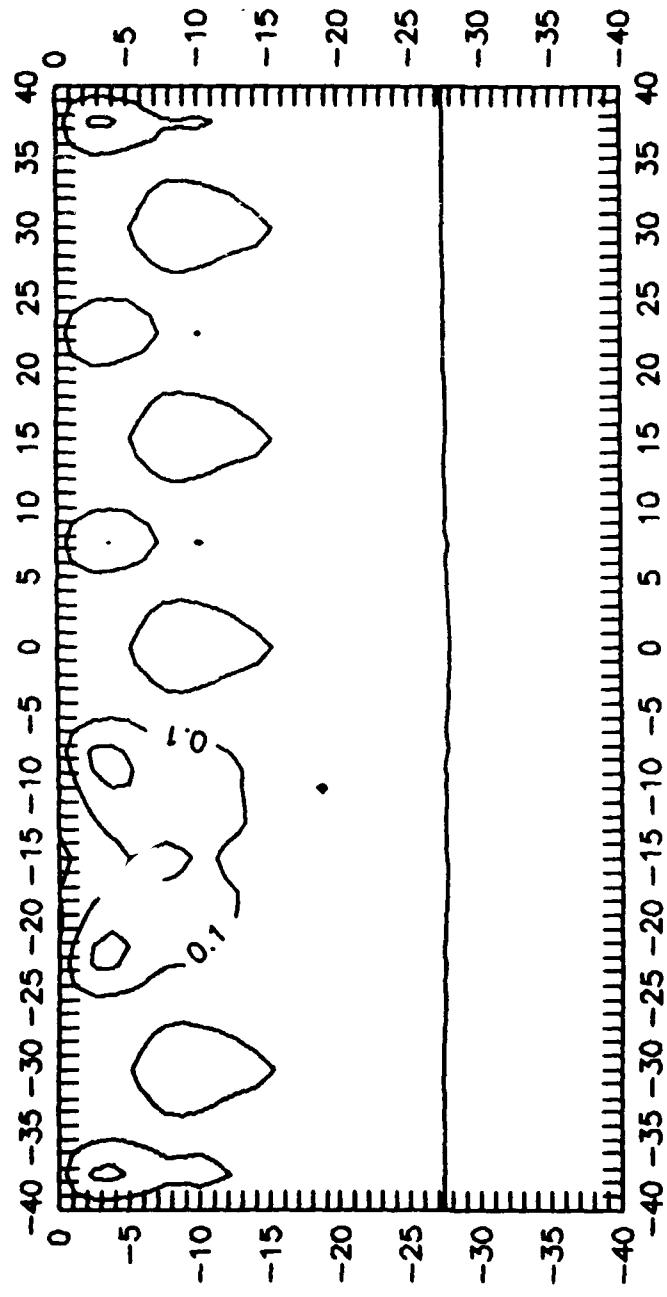
MOBILE 0.0 - Horizontal Deformation

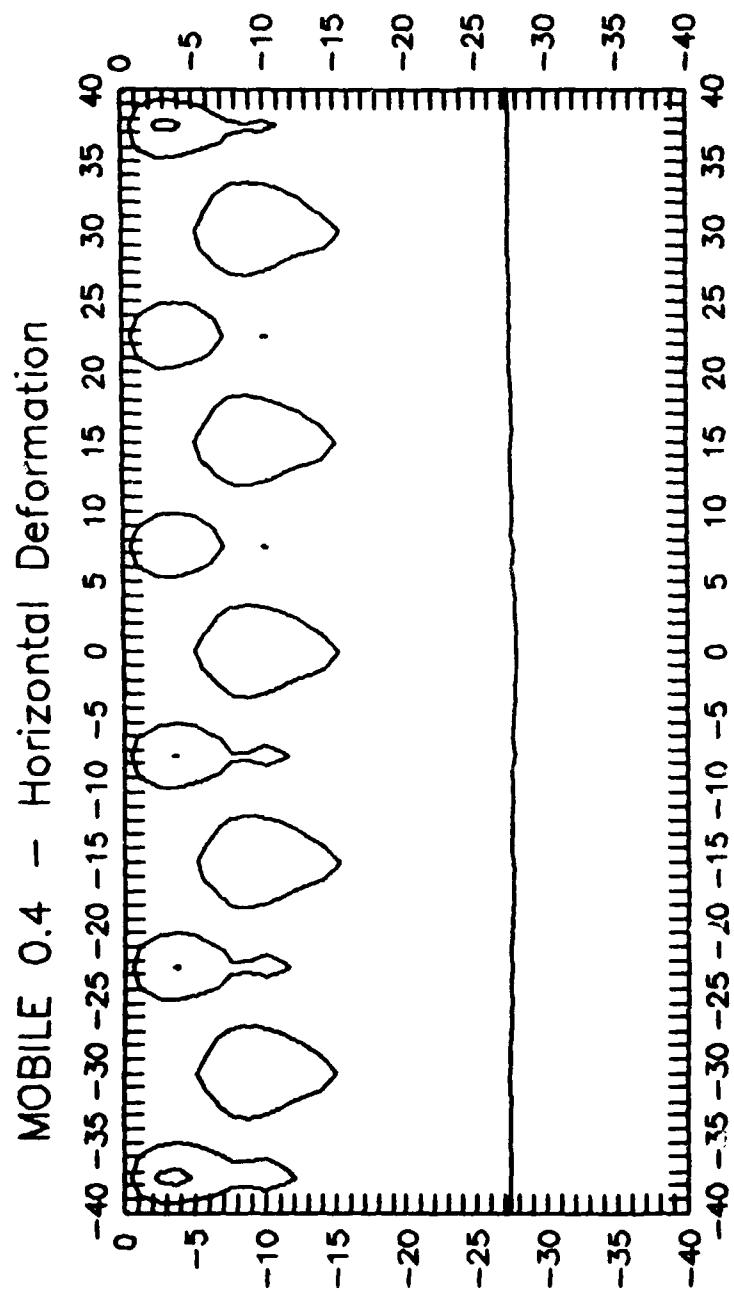


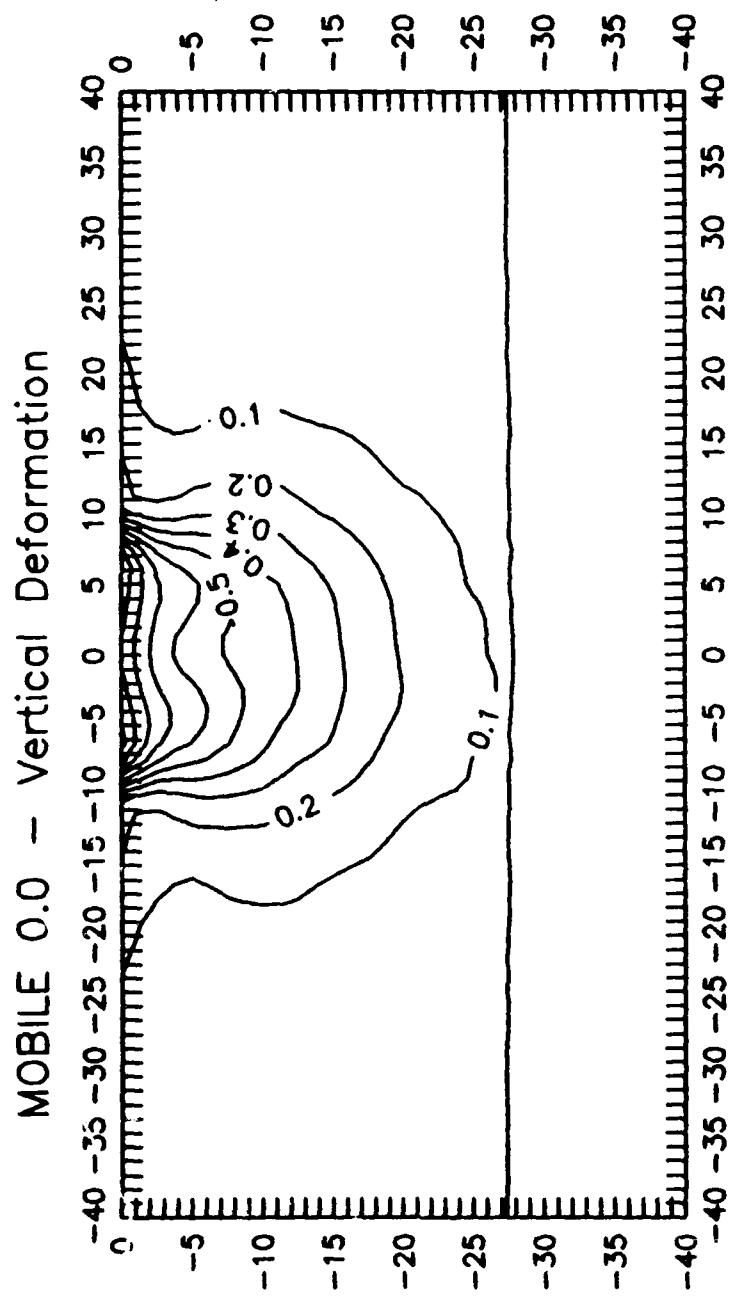


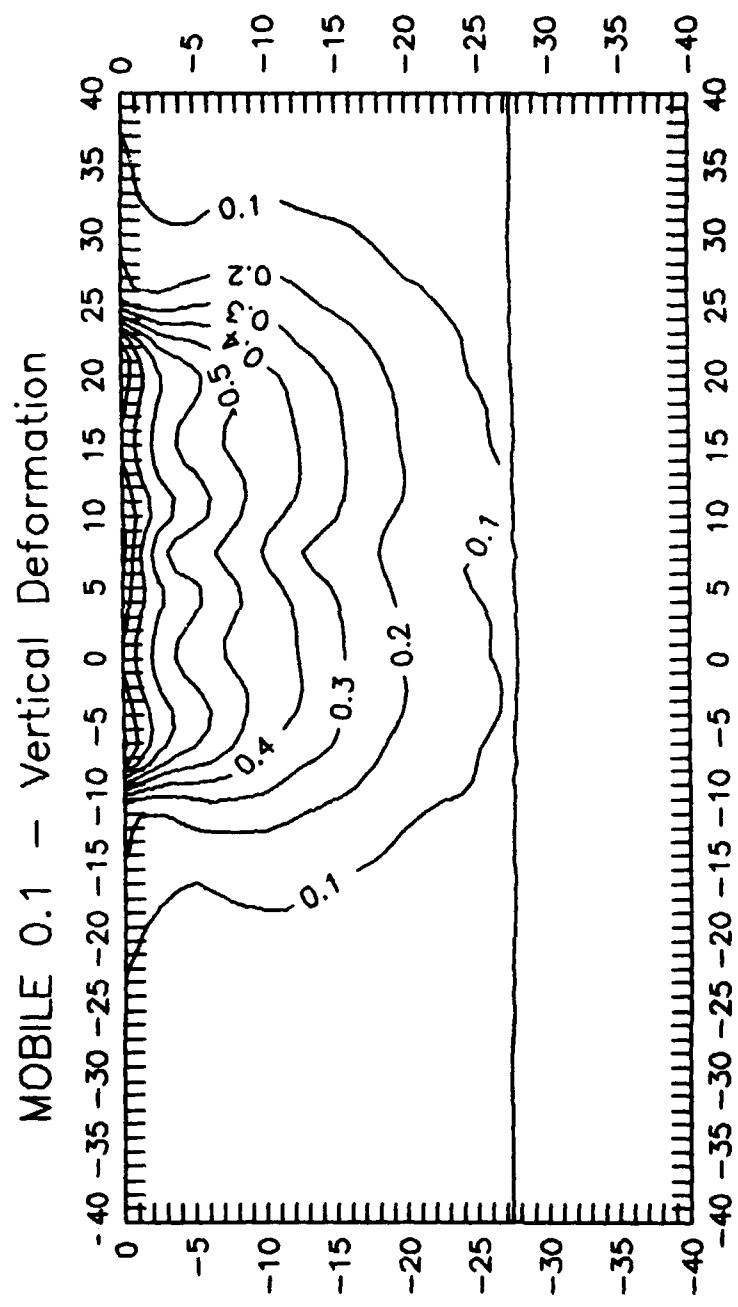


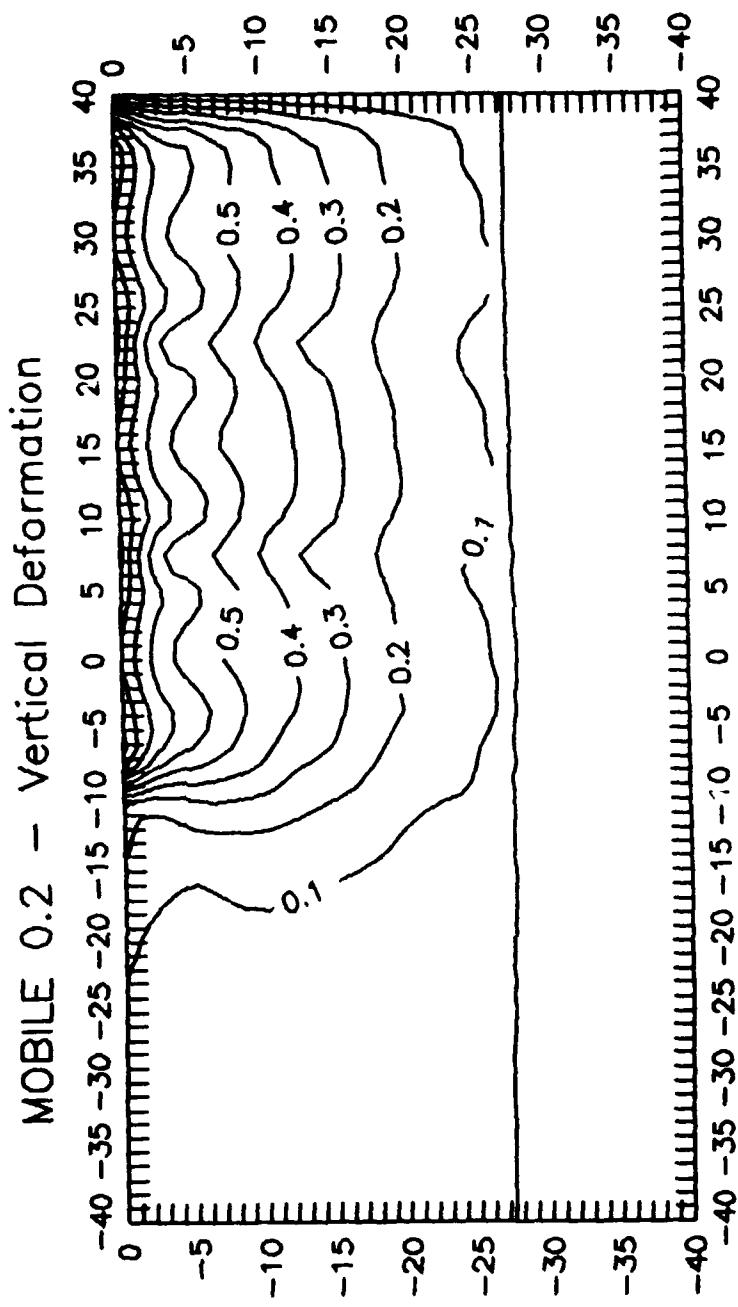
## MOBILE 0.3 - Horizontal Deformation

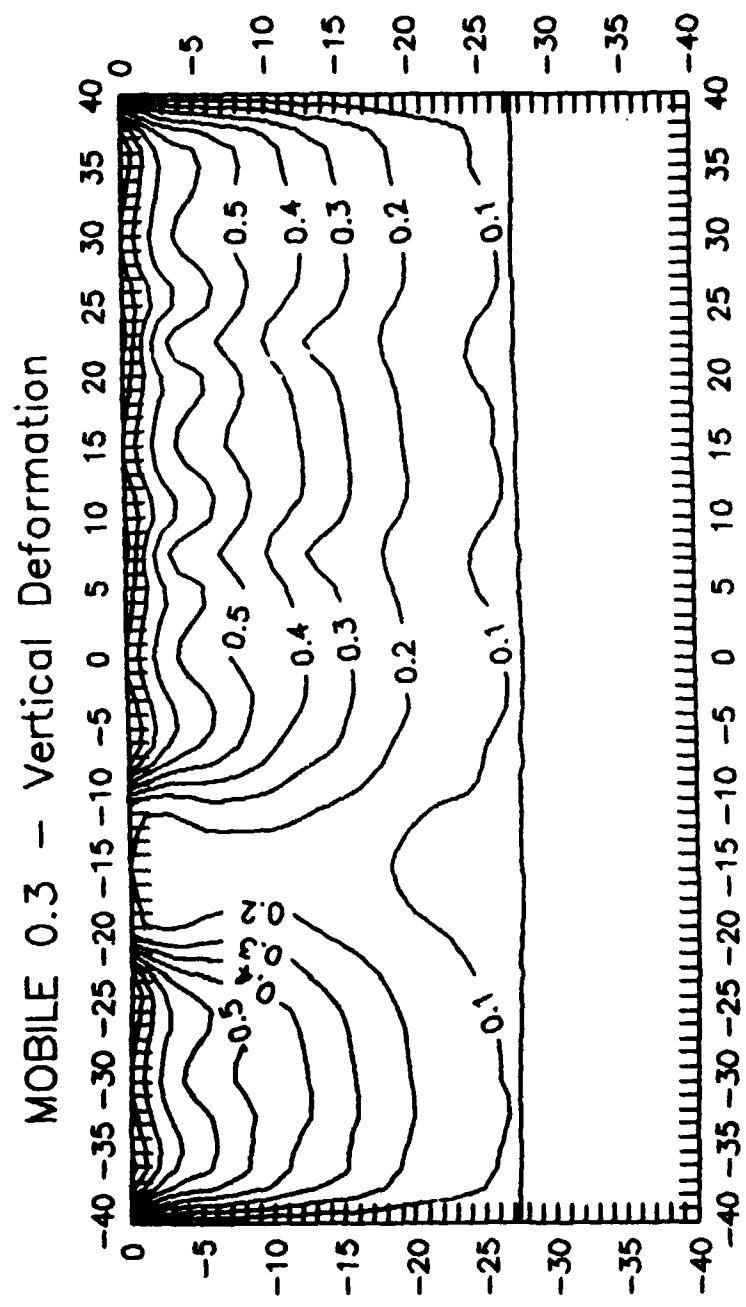


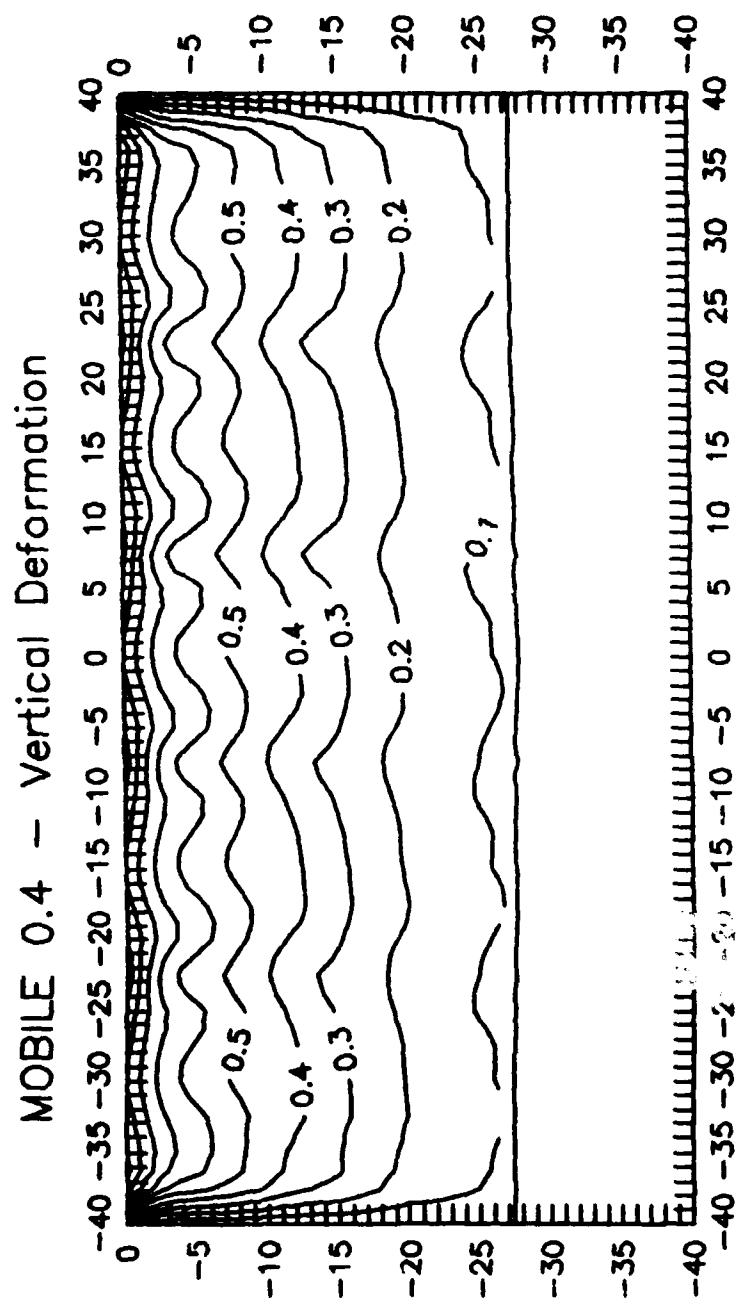


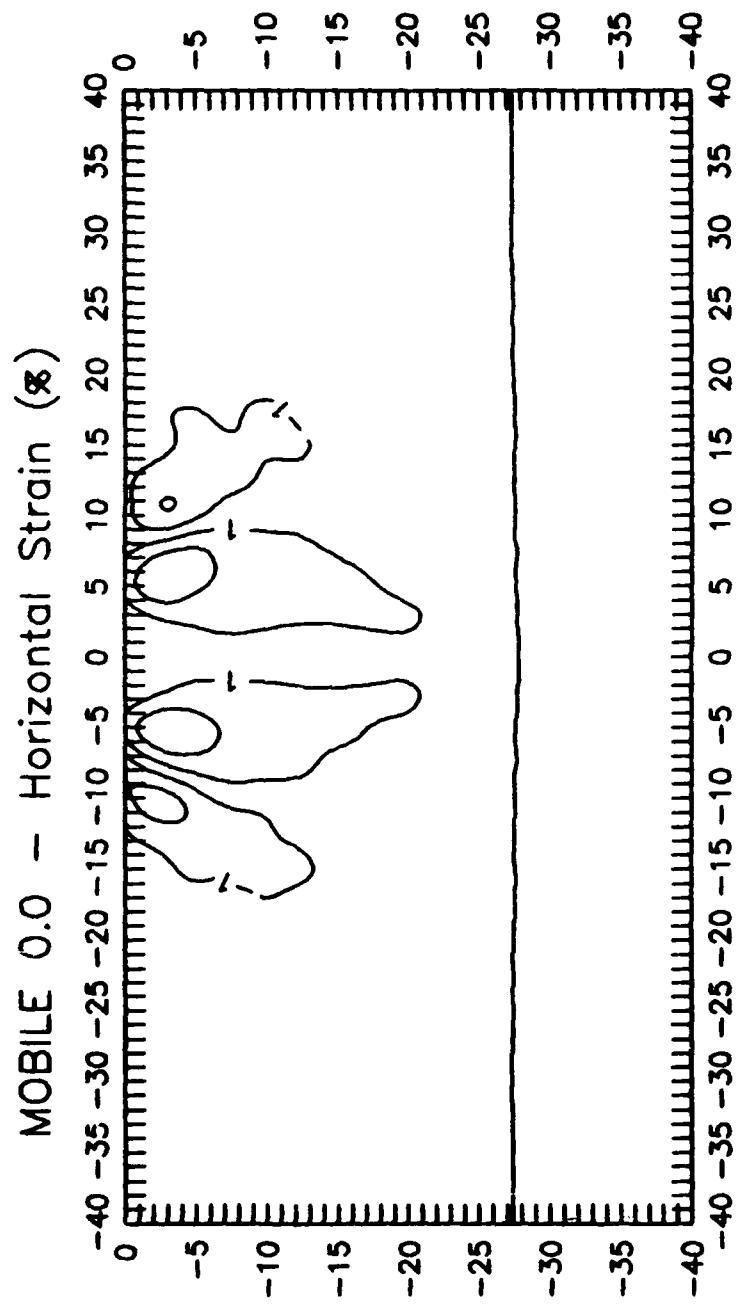


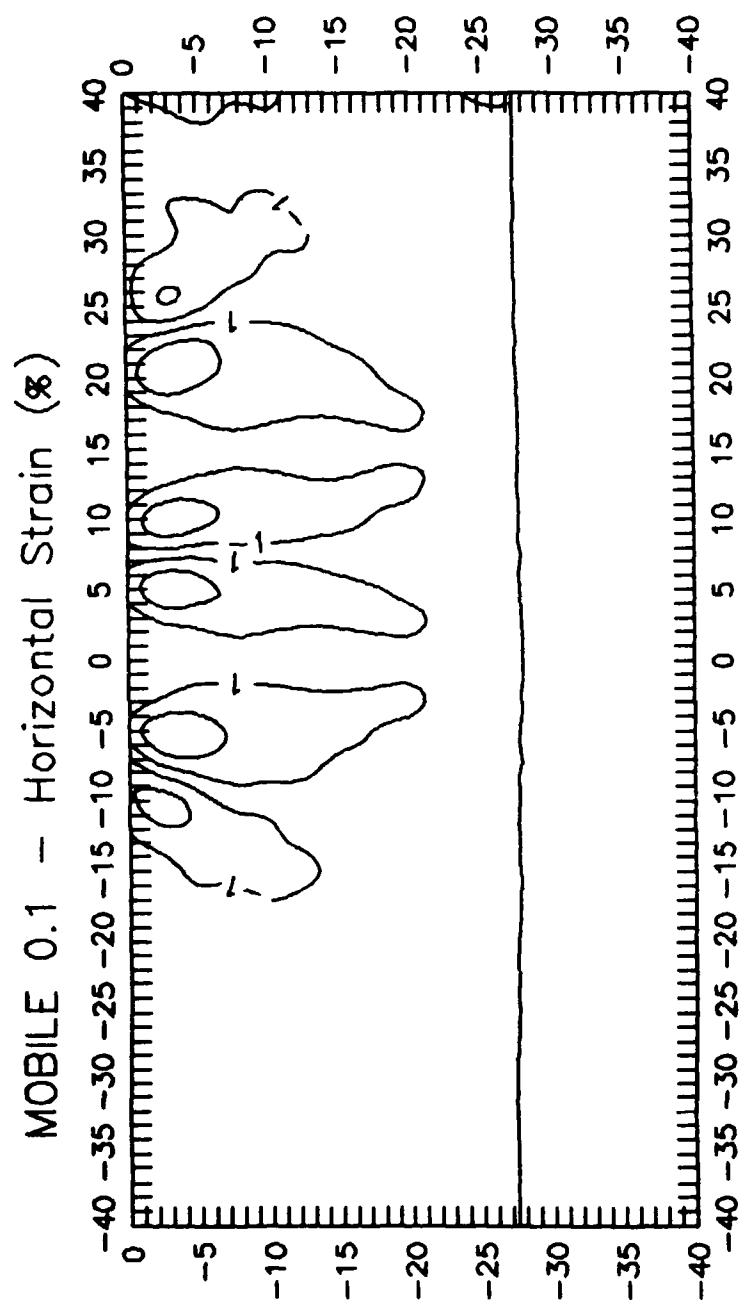


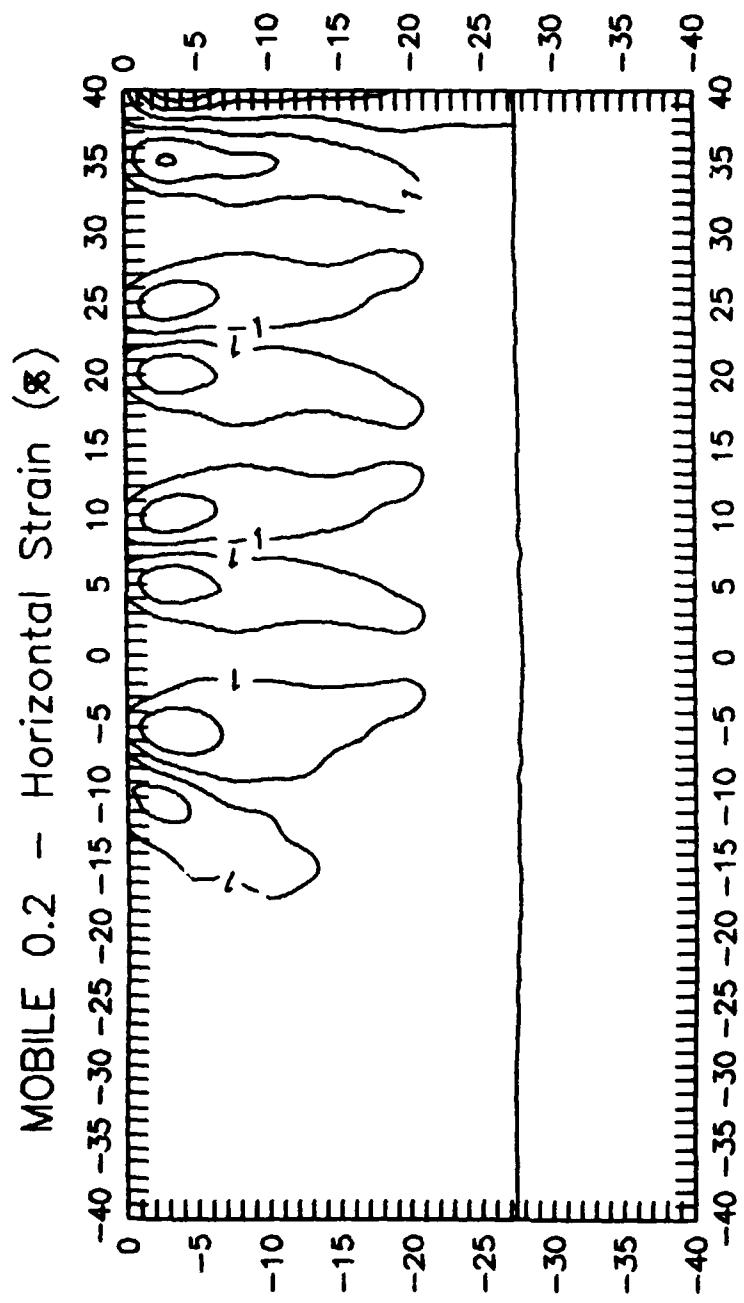


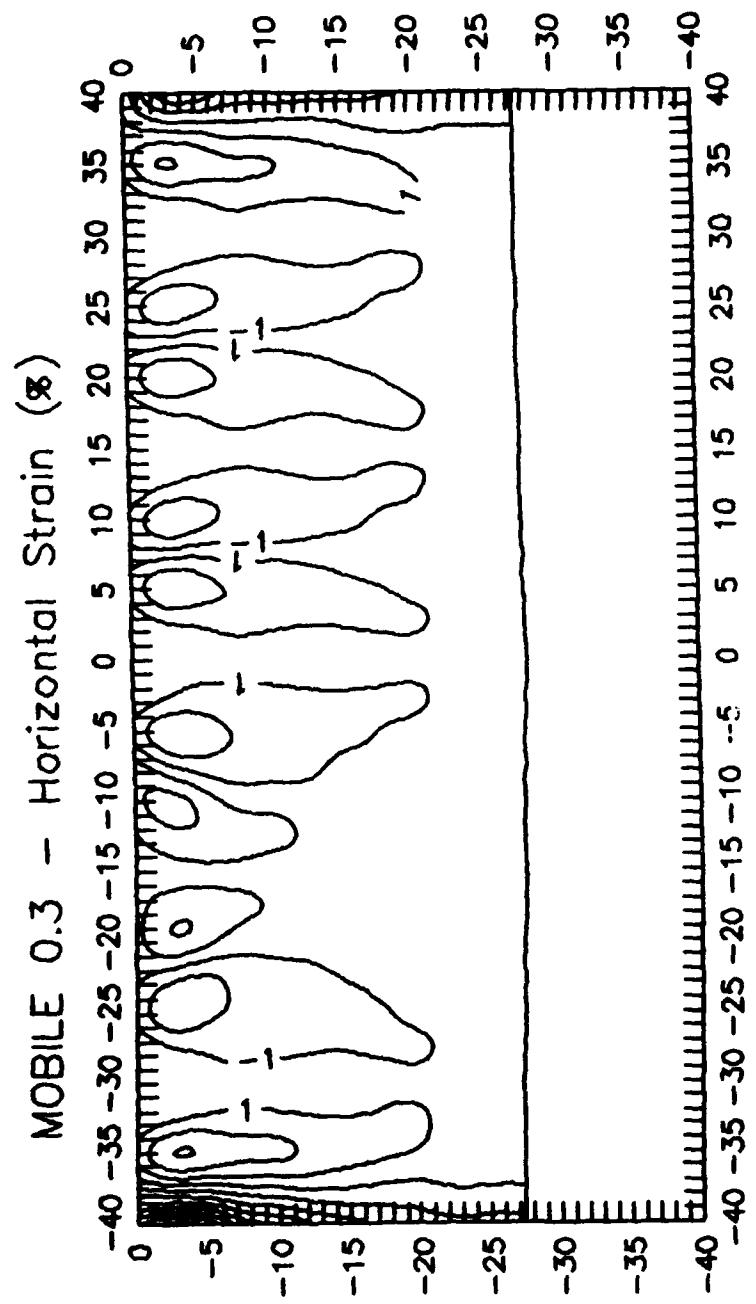


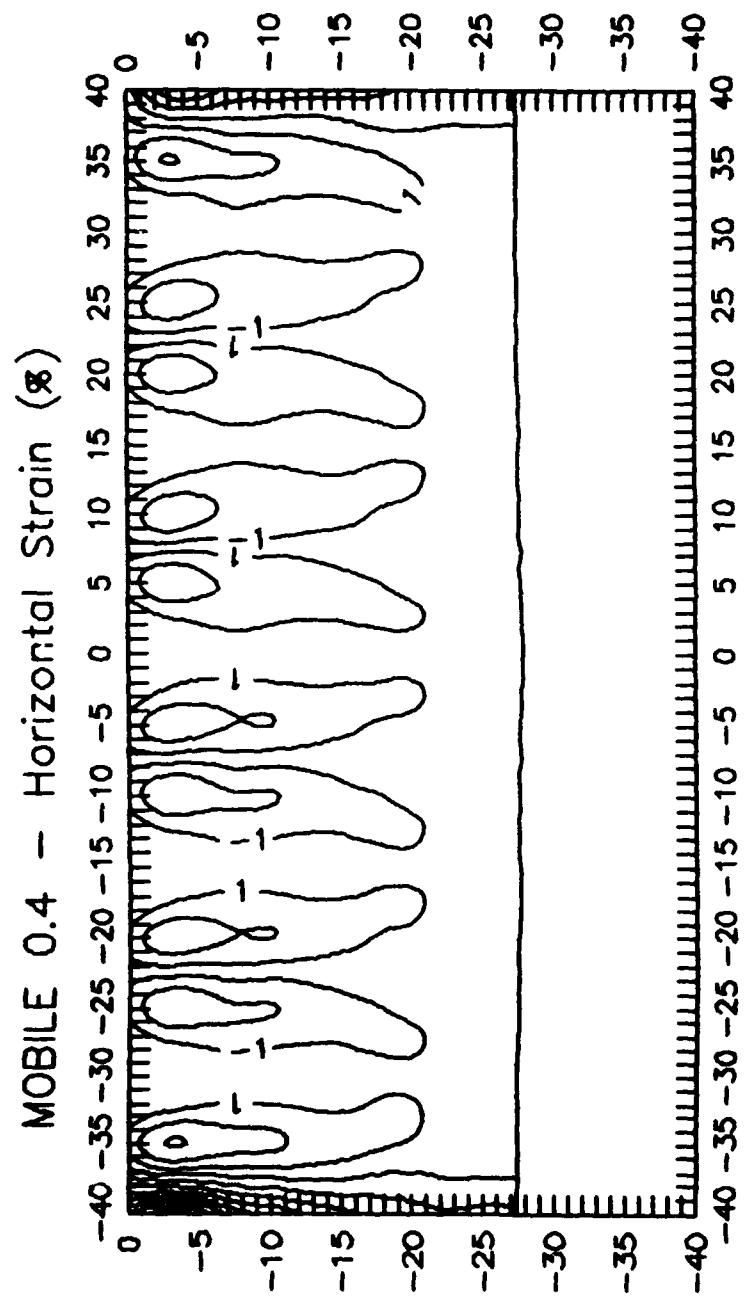


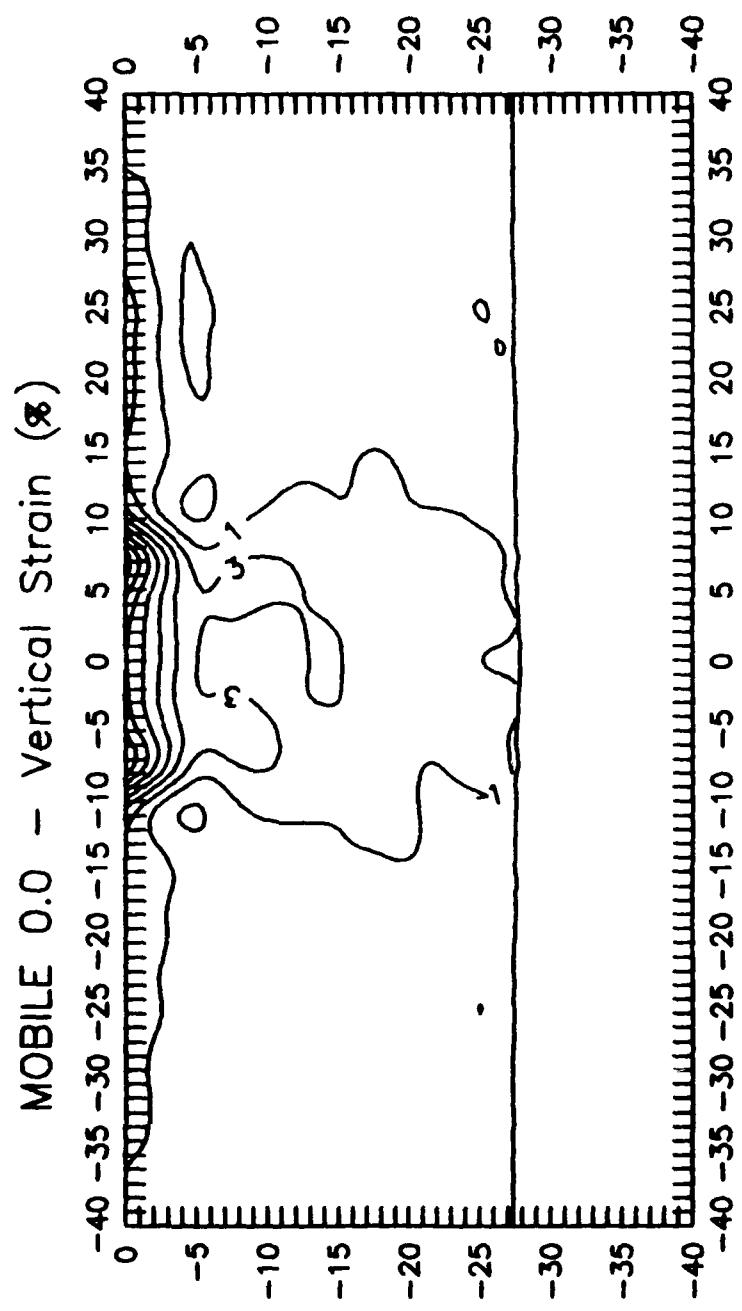


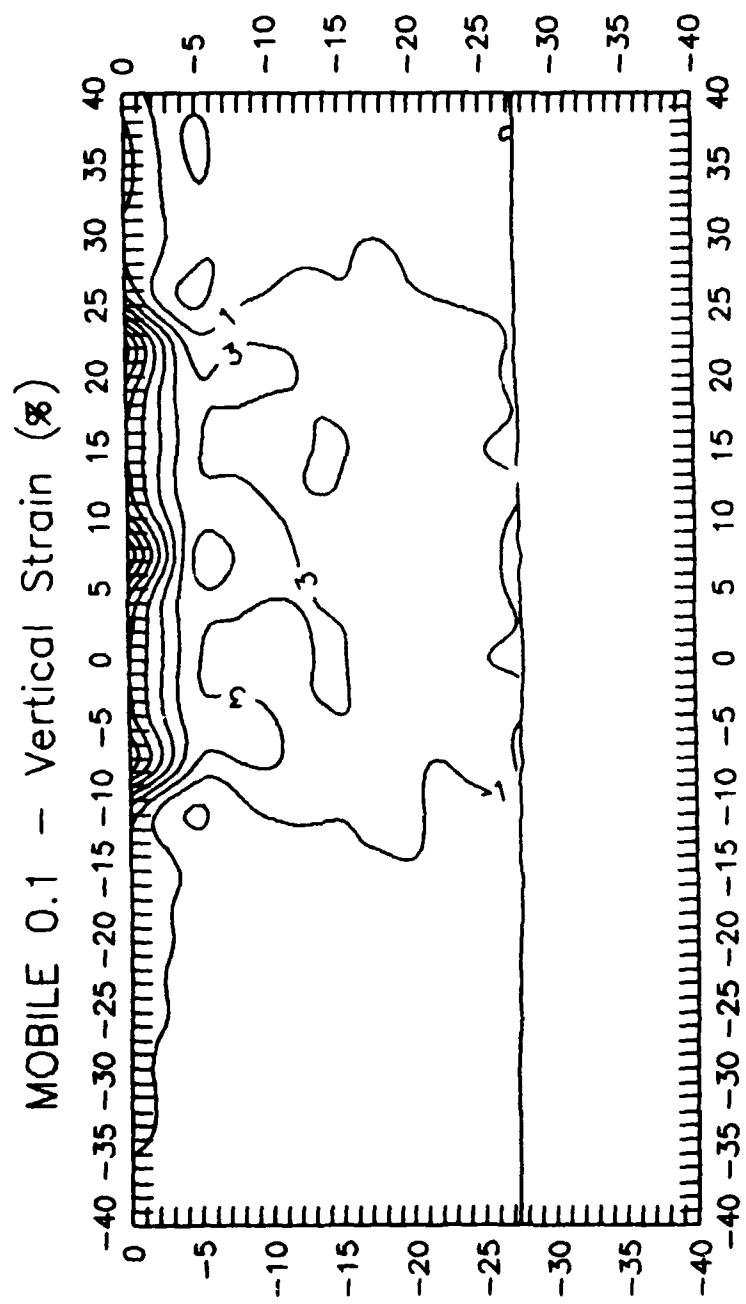


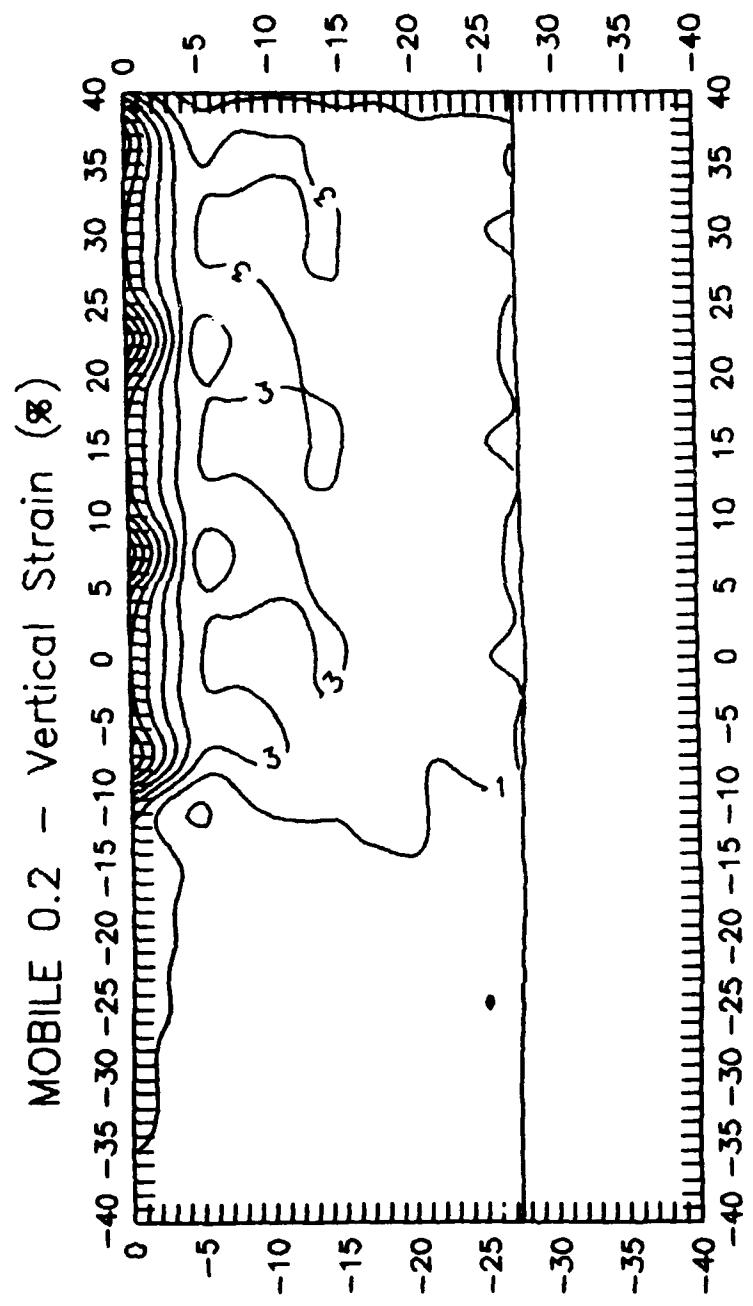


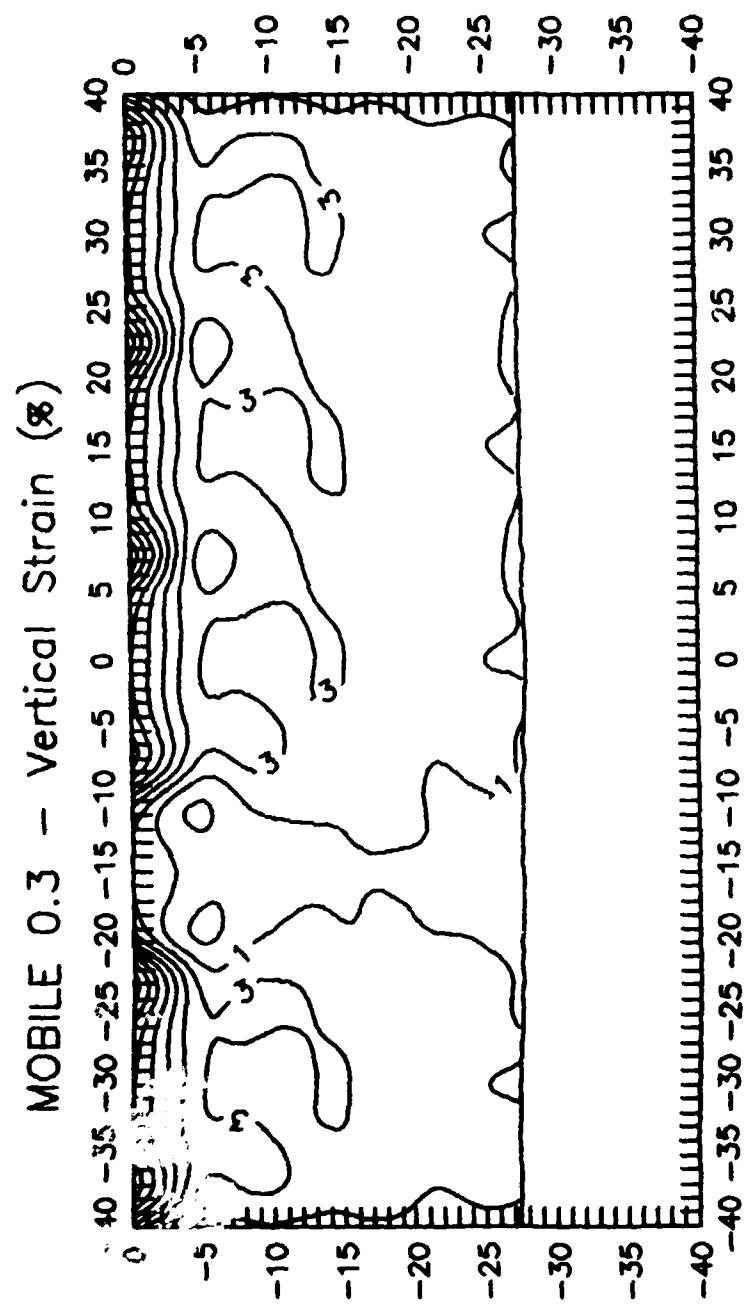


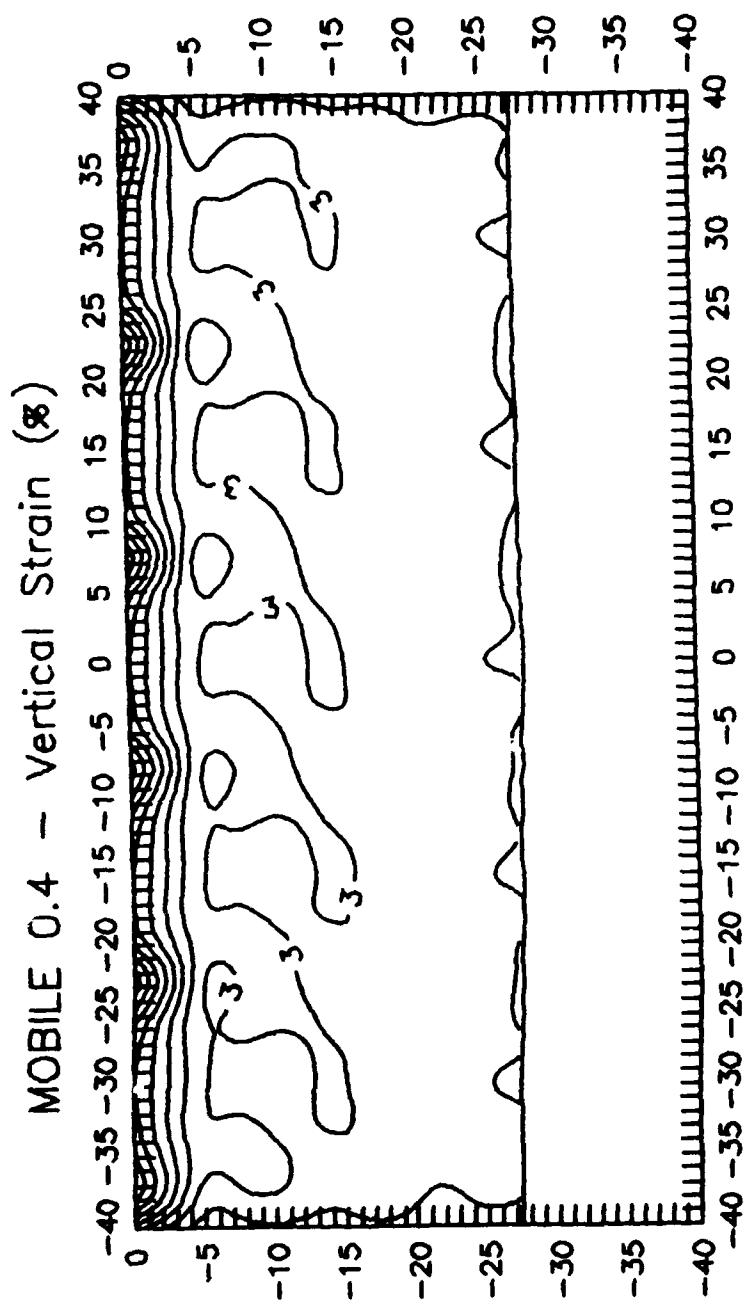


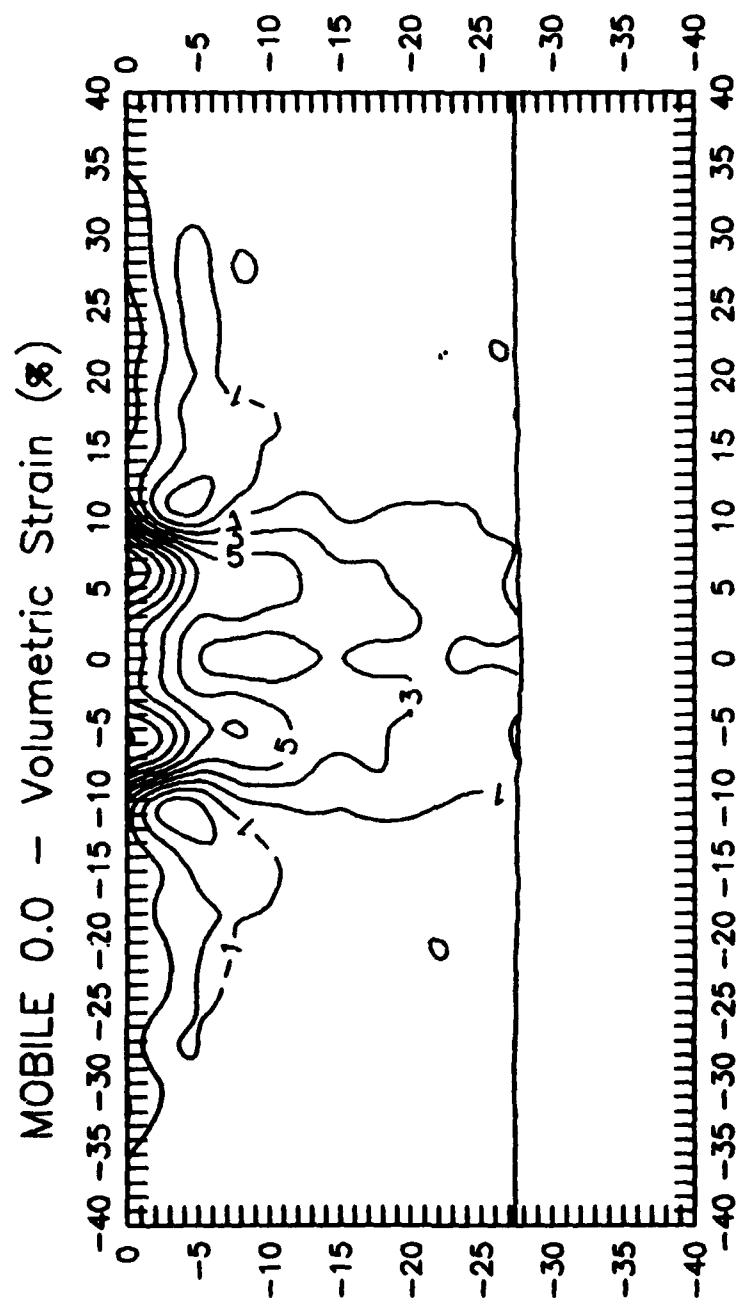


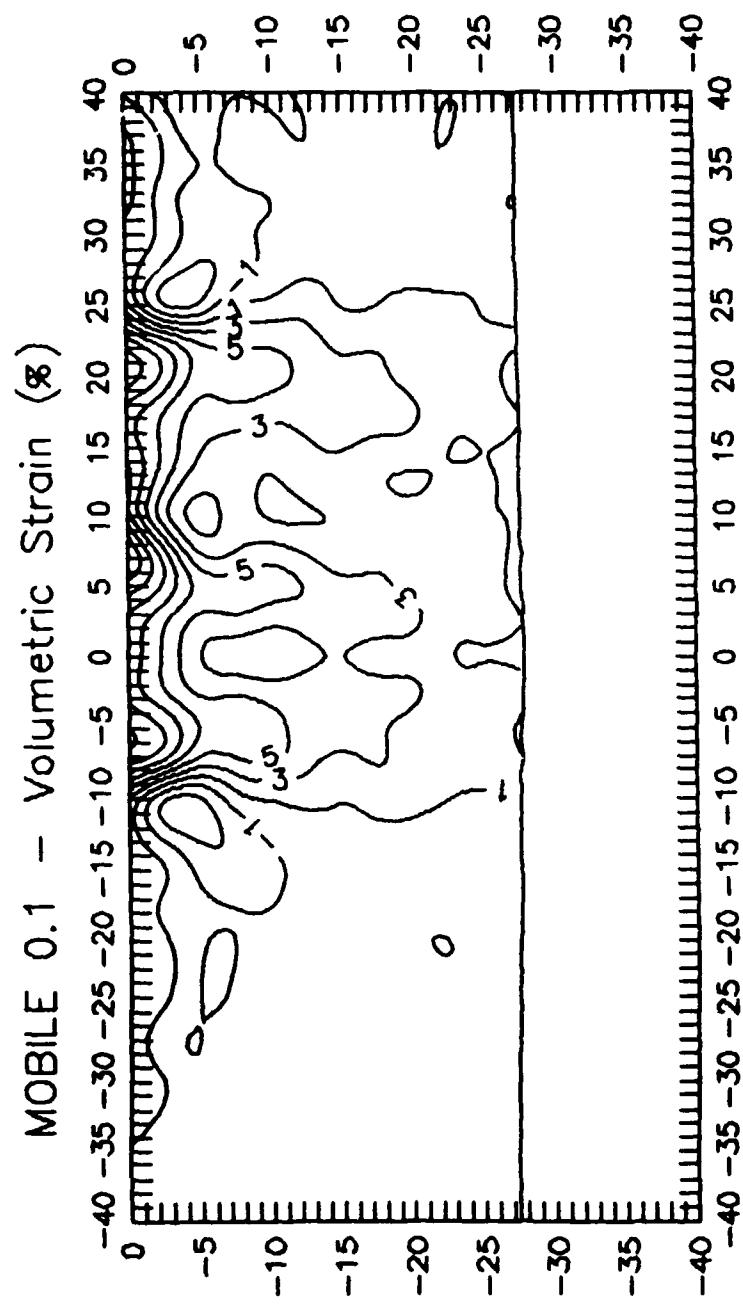


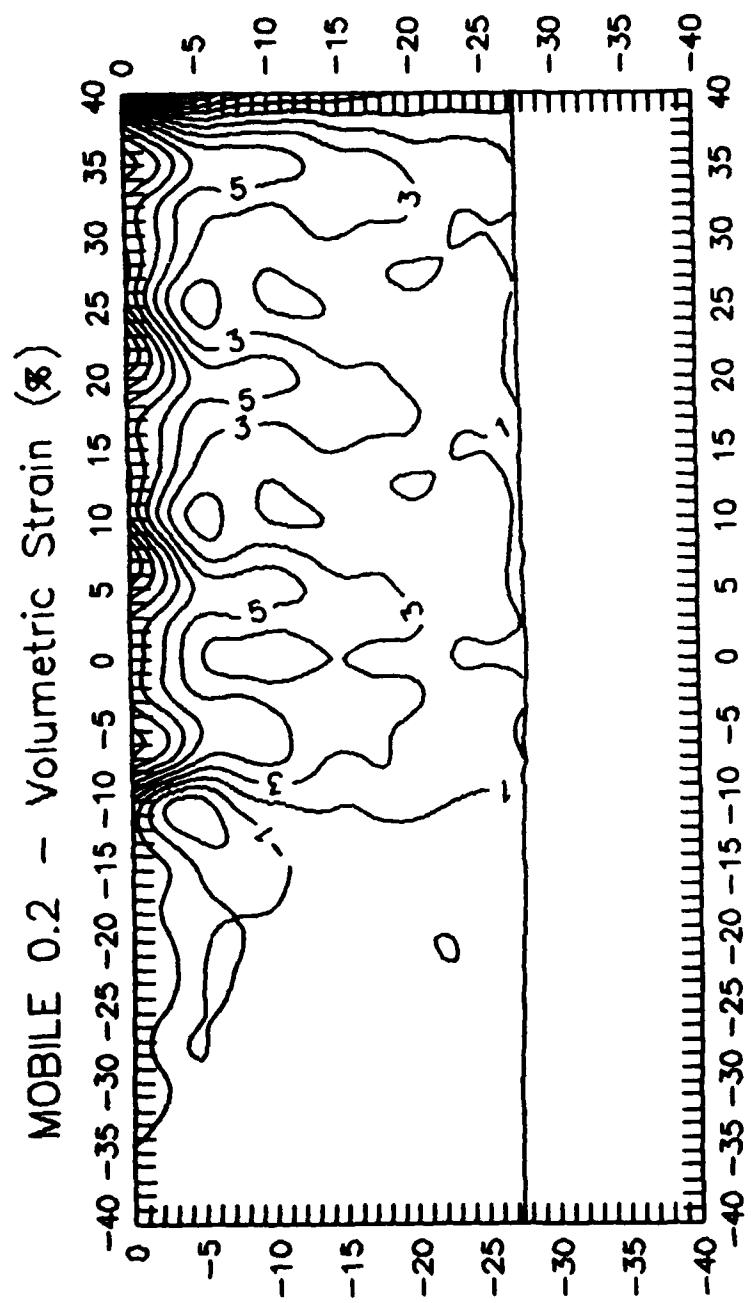


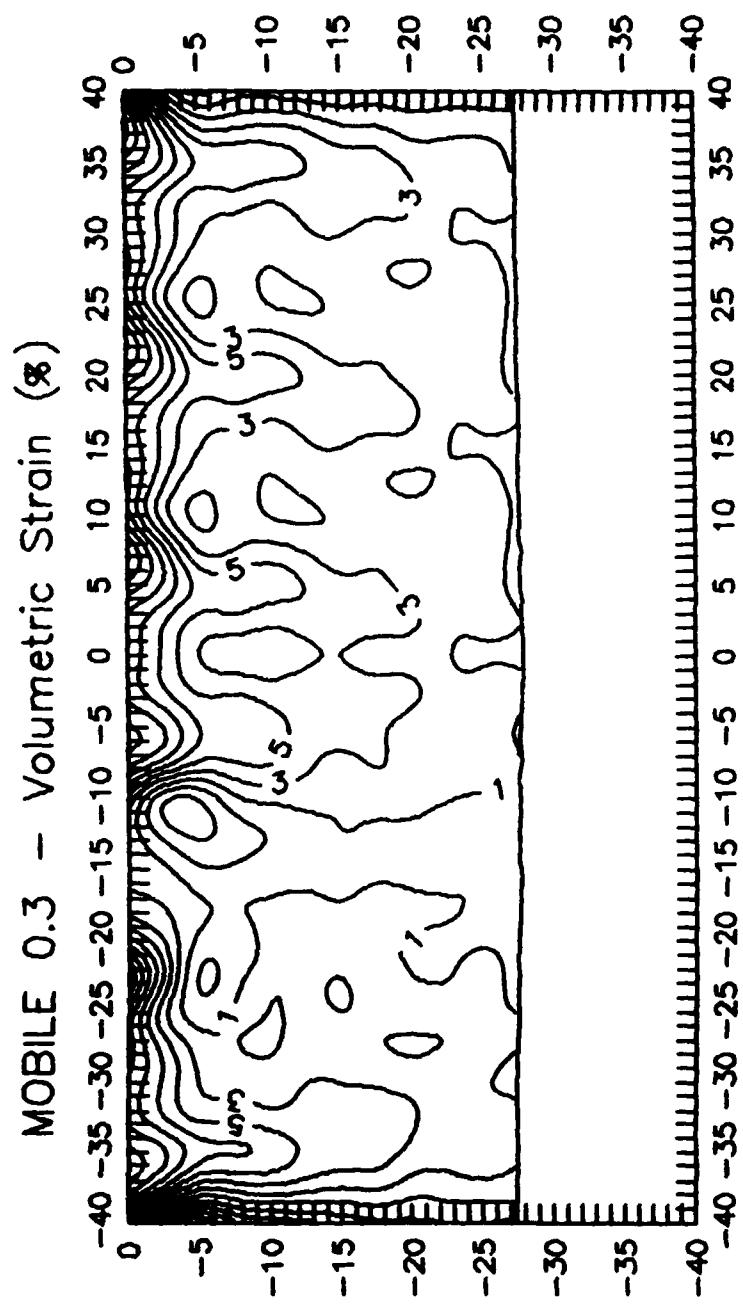


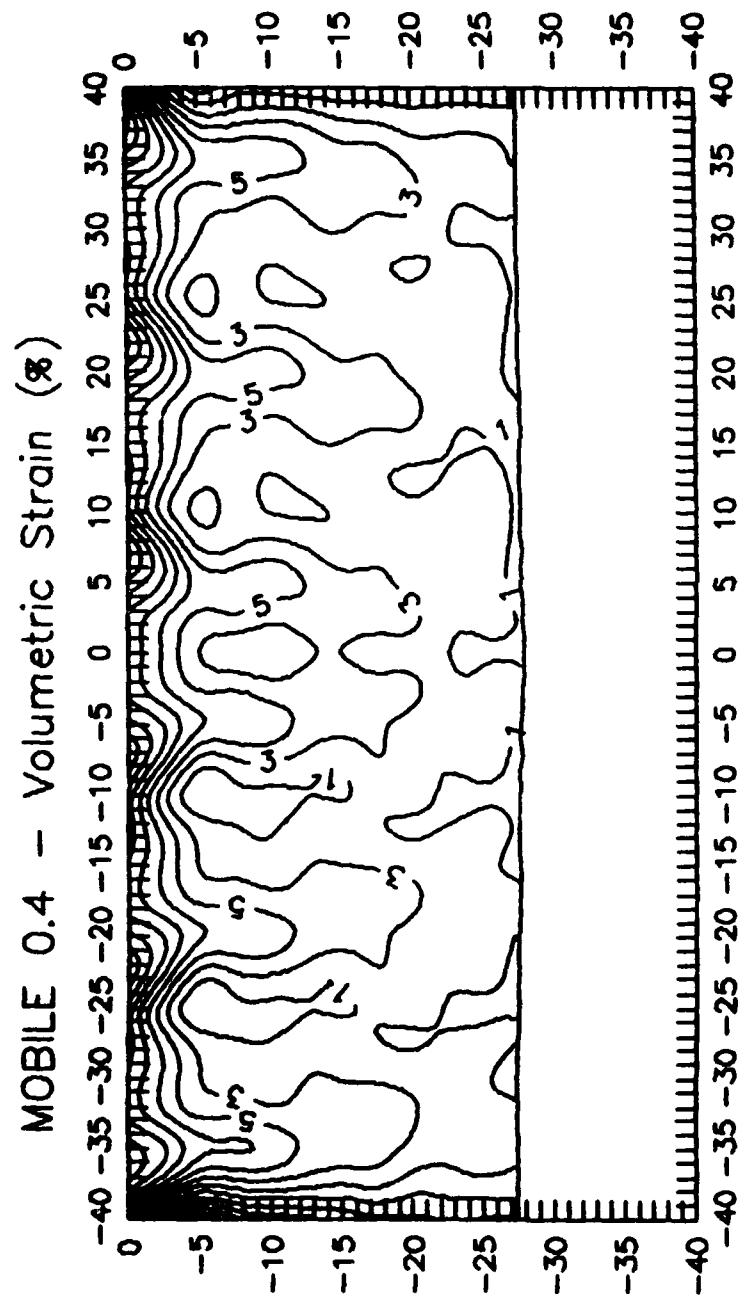


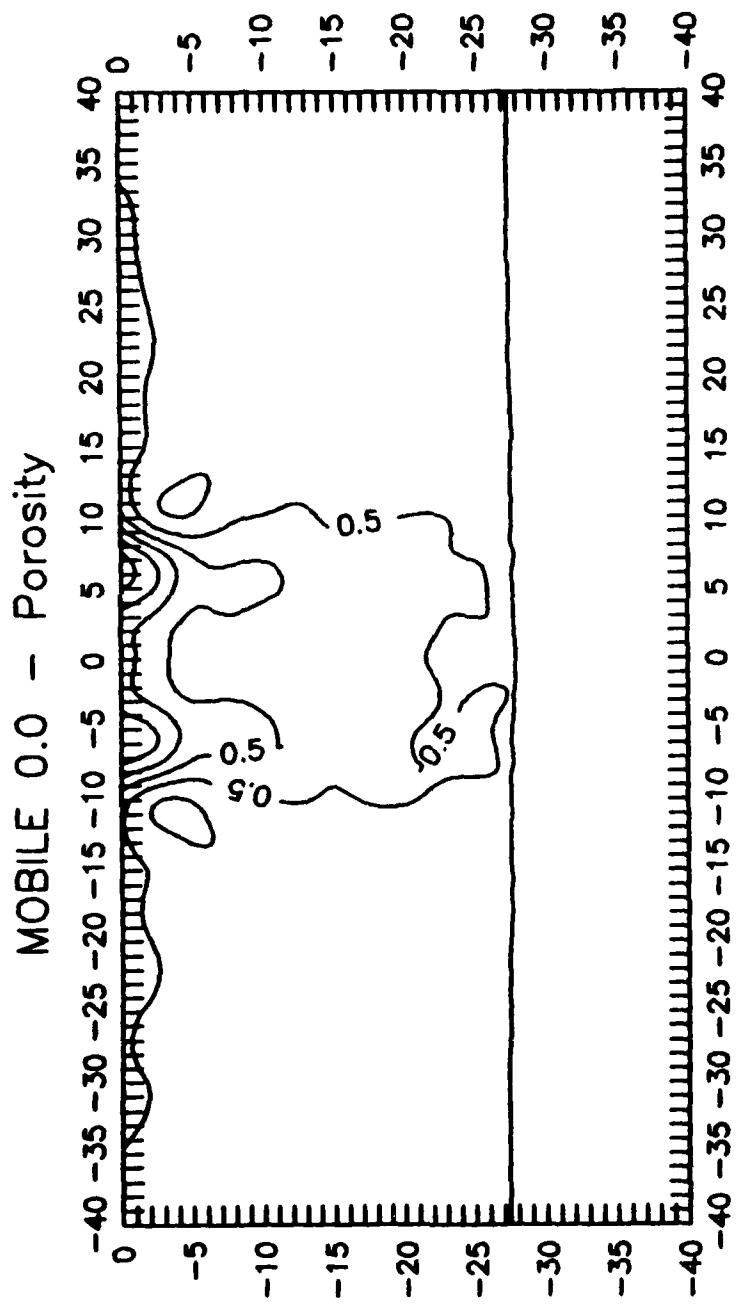


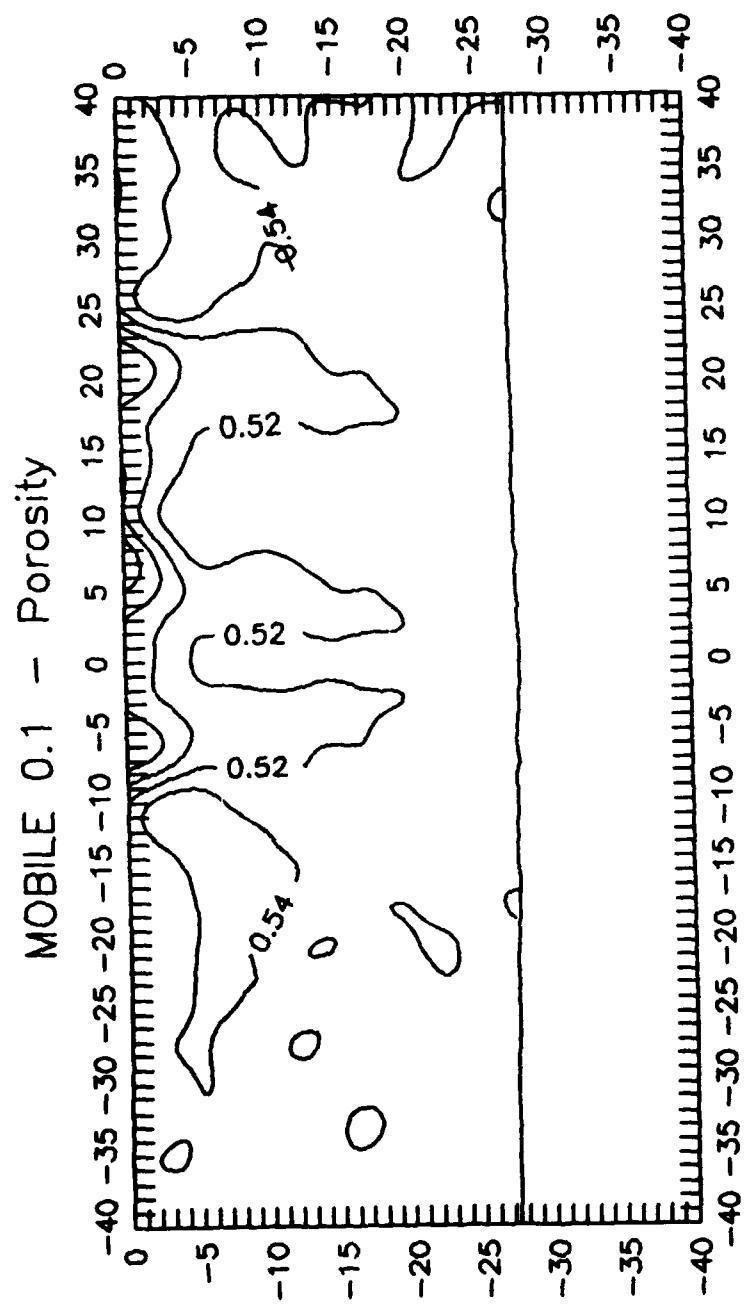


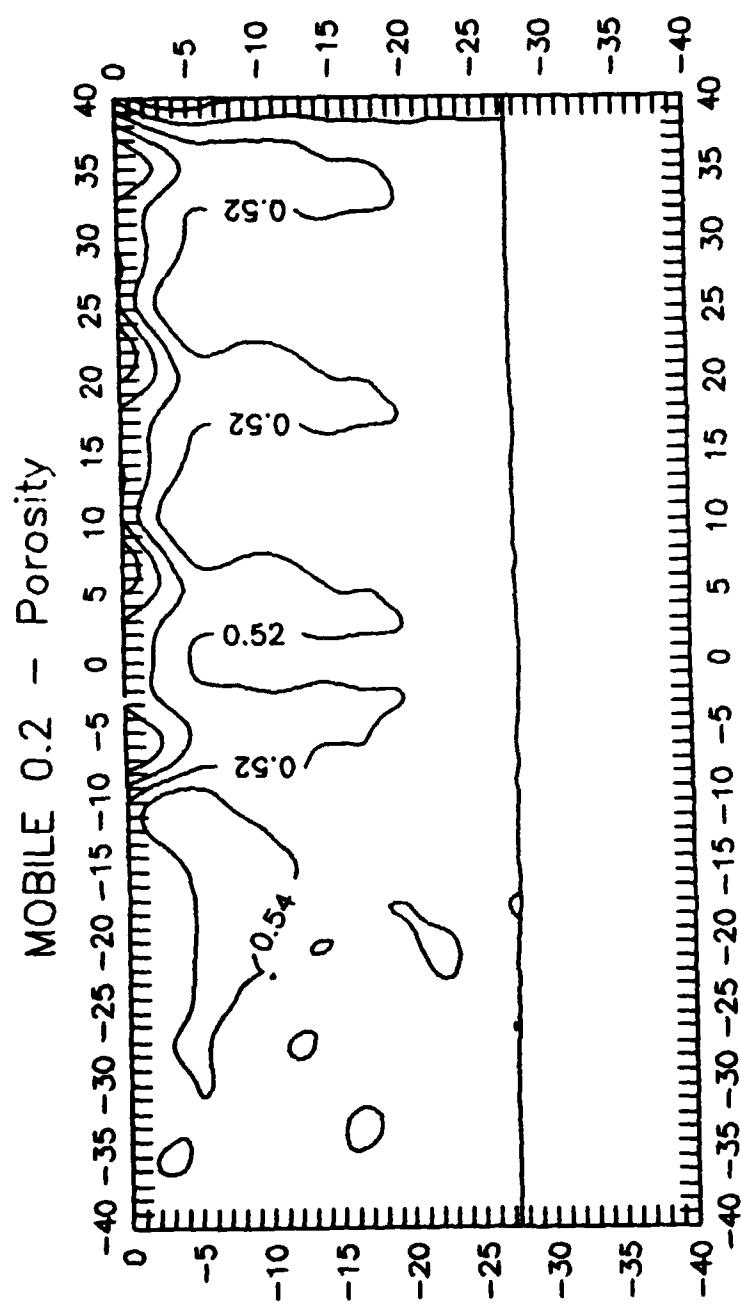


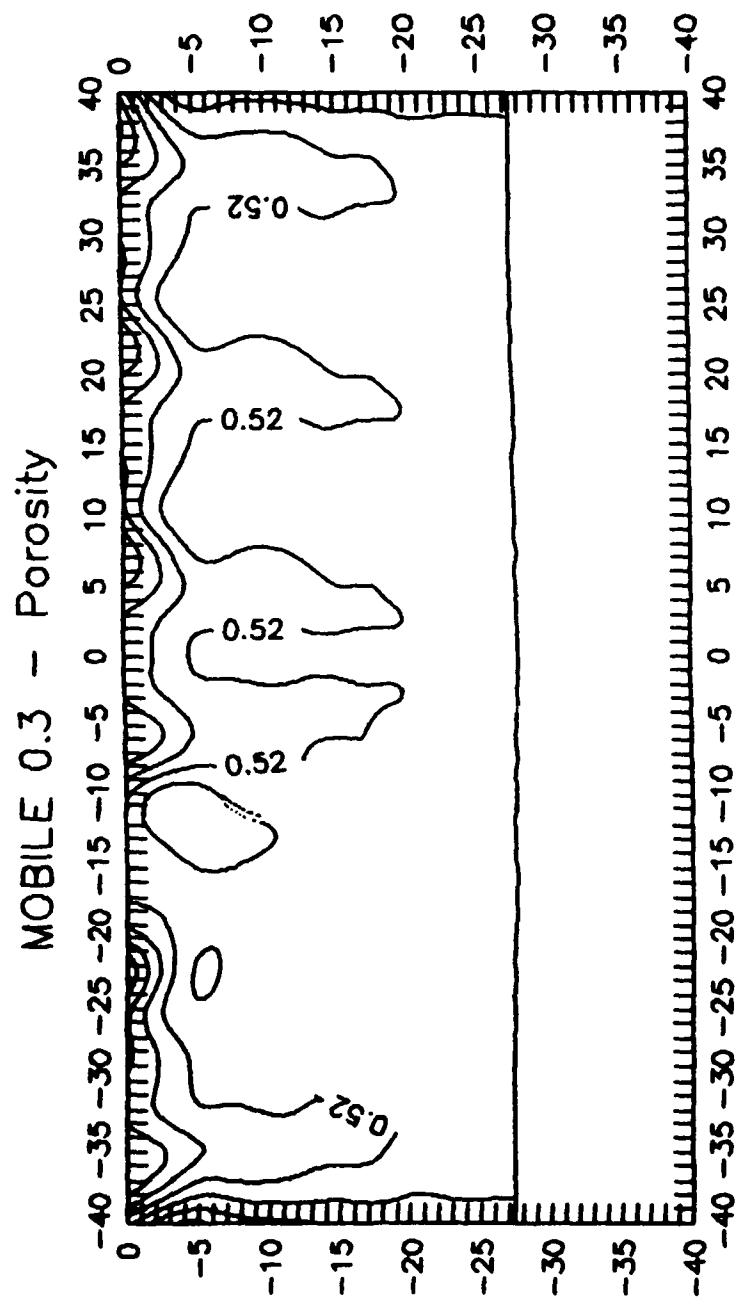


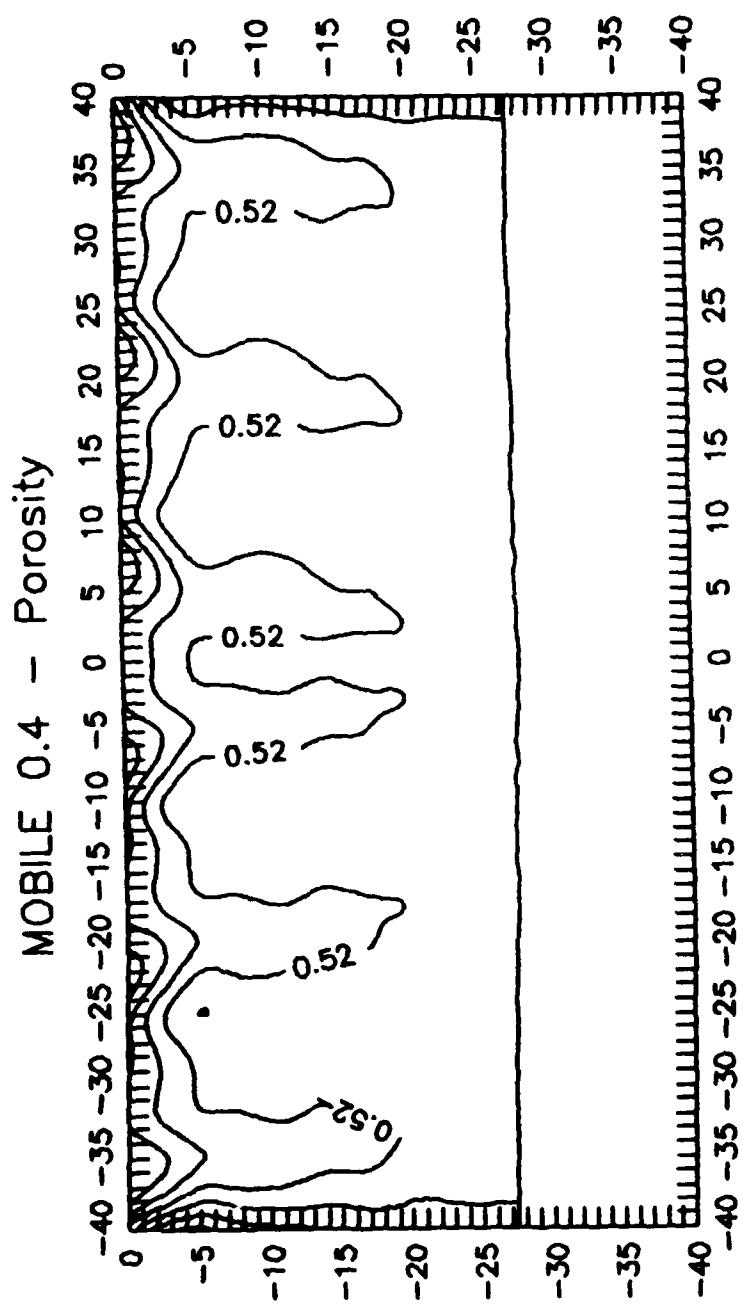






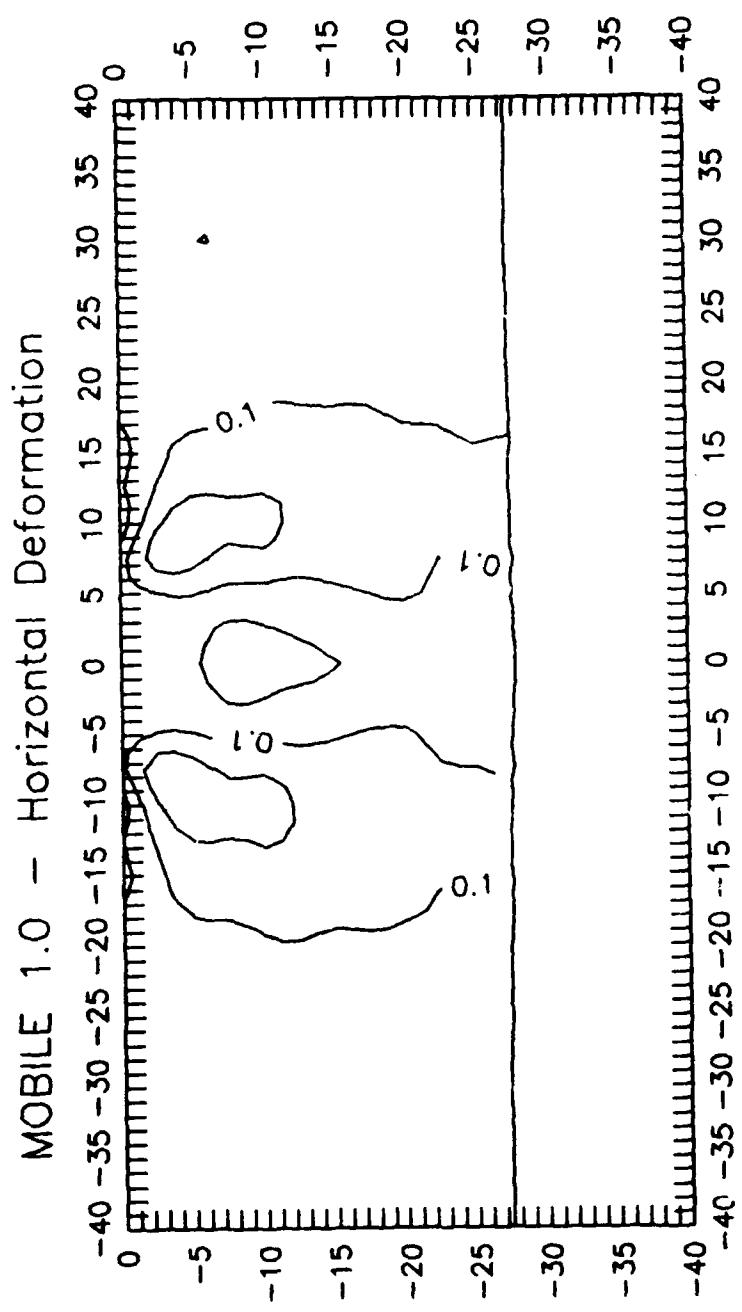


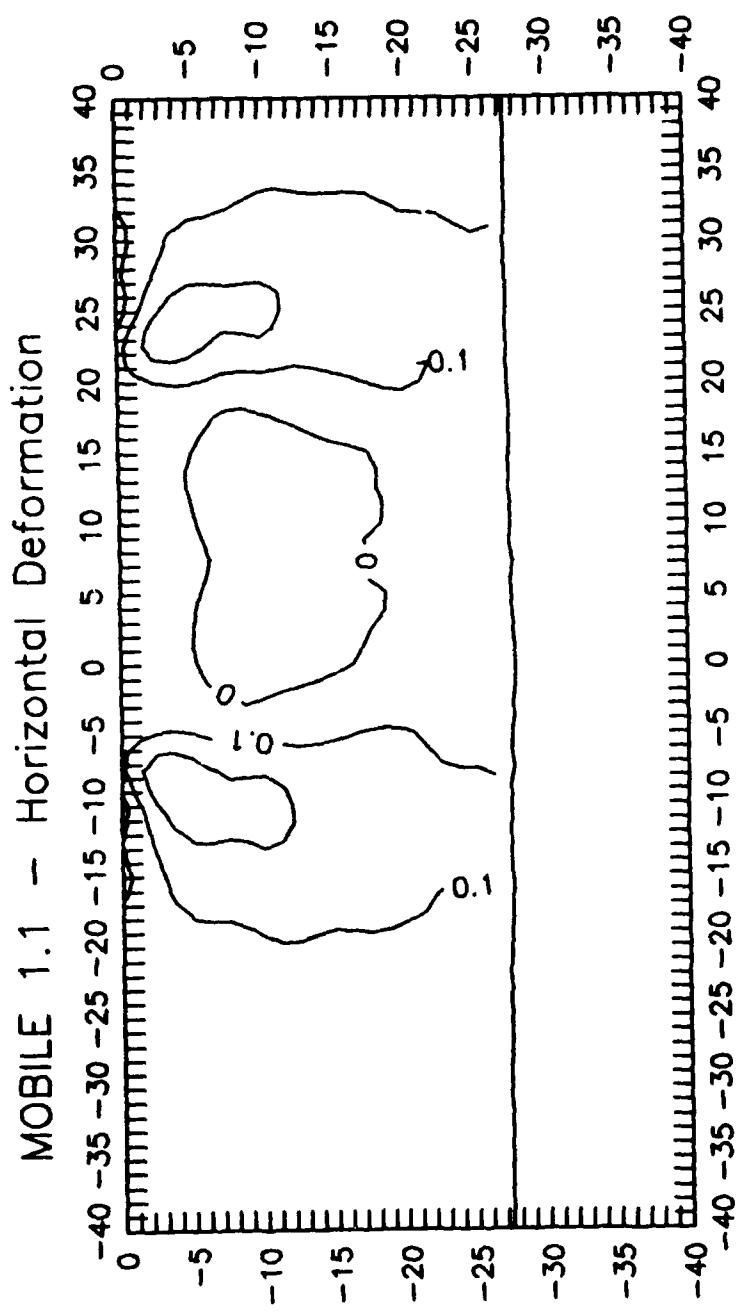




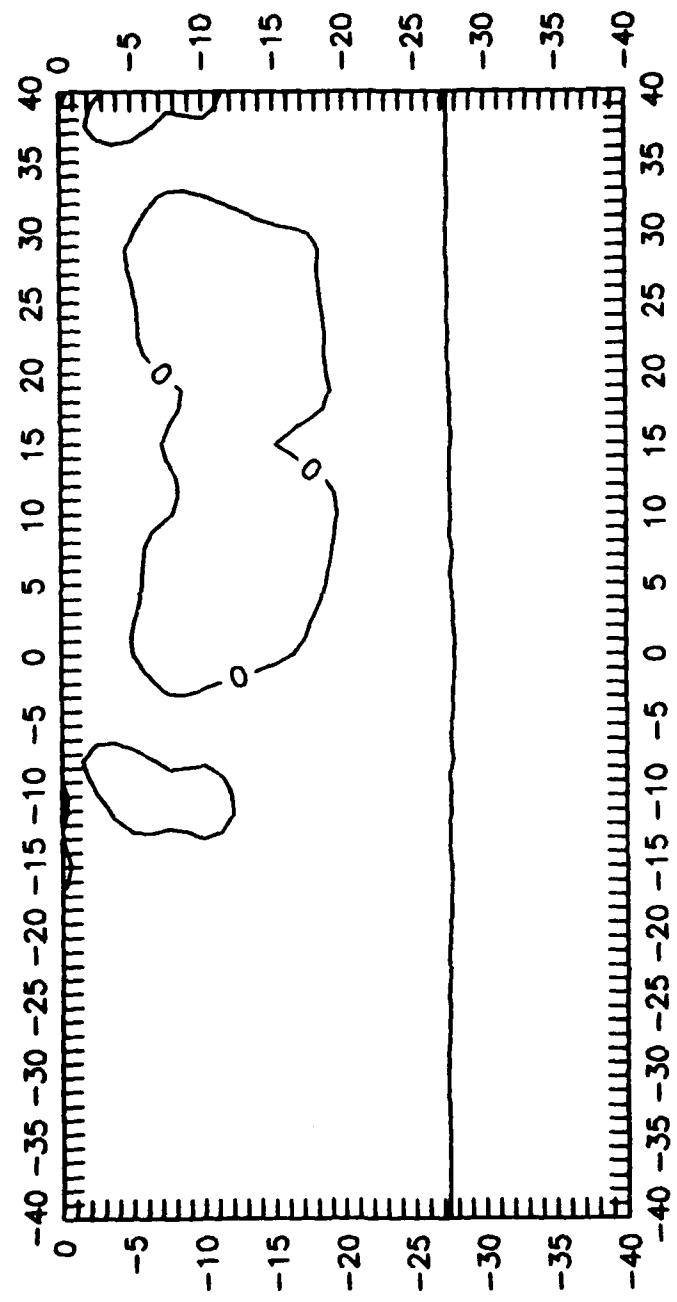
MOBILE SERIES

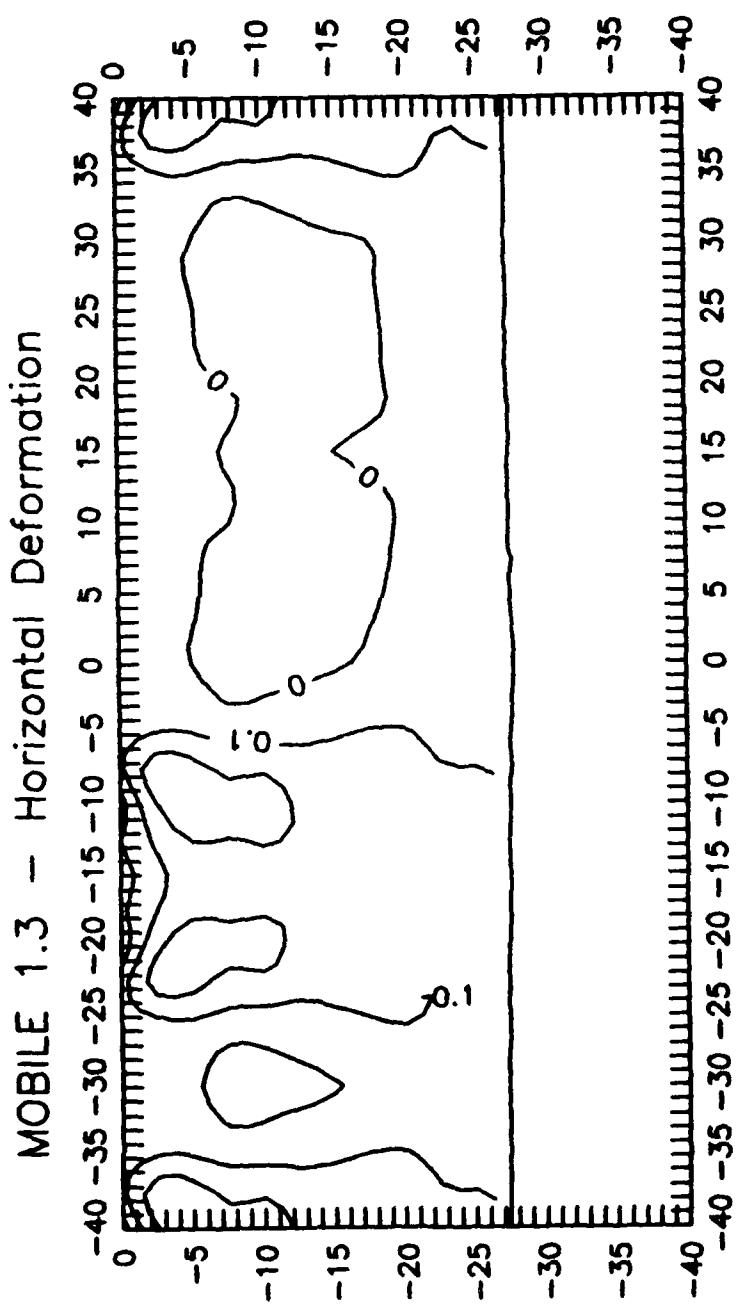
Load 35.0 kN/m<sup>2</sup>





MOBILE 1.2 - Horizontal Deformation





MOBILE 1.4 — Horizontal Deformation

